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Approaching capacity limits in photon-starved noisy optical communication

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ABSTRACT

We show that utilizing pulse position modulation and photon number resolving detectors together with techniques such as quantum pulse gating allows to approach ultimate quantum limit on optical communication capacity in the presence of background noise in the weak output power regime. This shows that for communication over long distances by means of current existing technology it is possible to attain optimal performance, limited only by laws of quantum mechanics.

Keywords: Photon-starved communication, pulse position modulation, photon information efficiency, Holevo's bound, noise rejection, photon counting

1. INTRODUCTION

Switching to the optical domain promises great advantages in deep-space communication over traditional methods. From the physical point of view the main benefits are, as compared to radio communication, vastly increased bandwidth, and the existence of laser sources. The latter is particularly important for large distances as it allows to significantly reduce diffraction losses caused by beam divergence. The optical communication systems were or will be tested on several occasions, including LEO to GEO transmission,[1] ground to GEO,[2] lunar missions,[3] and missions further in the Solar system.[4]

A standard approach to deep-space optical communication employs pulse position modulated (PPM) signals.[5] PPM format encodes information in a position of a light pulse in an otherwise empty train of time bins. A direct detection receiver is then used to identify the position of the light pulse. It is known that in the noiseless scenario such a strategy allows one to approximately achieve the ultimate bound on communication capacity imposed by the laws of quantum mechanics in the low power regime[6,7] typically encountered in deep-space communication. This is because in the absence of noise direct detection unambiguously identifies position of the light pulse, and one can use a simple decision rule where the only source of errors are erasure events in which the detector did not click when the pulse was present. However, in the presence of noise, such as background light or dark counts, the detector may also fire in time slots that are not occupied by the pulse. This additional source of errors makes the simple decision rule strategy problematic, as events in which there are multiple detector clicks within a single PPM frame can arise. As a result, in the simplest decoding scenario, the quantum bound on the capacity not only cannot be saturated, but also the transmission rate becomes proportional to the square of the average received signal power, which makes it very small in the photon-starved regime.[8,9] One can improve this result by employing a more general decision rule based on soft decoding and attain a linear scaling of rate with the average received power, however, a potentially large gap still remains to be overcome to approach the quantum bound.[9,10]

In this work we show that the quantum bound on communication capacity in the low-power regime in the presence of noise can indeed be attained by employing PPM together with photon-number resolving detectors. To approach the capacity bound, it is necessary to utilize a soft decoding strategy and appropriately large PPM orders or, equivalently, the power of the signal light pulse. Importantly, we find the approximate value

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of the optimal power of the signal pulse and show that it scales proportionally to the logarithm of the inverse of the average signal power. Crucially, we assume that one can efficiently discard all the overlapping temporal orthogonal modes except the signal mode, which can be performed by means of recently developed methods such as quantum pulse gating. Our results point to the possibility of improving the communication capacity in the photon-starved regime, particularly important for deep-space communication, with the use of already existing or being developed techniques.

This paper is organized as follows. In Sec. 2 we review the capacity limits for optical communication links, and in particular we discuss the ultimate quantum bound on the information transmission rate. In Sec. 3 we outline the model we consider in this work. Sec. 4 presents the results we derive for PPM in the photon-starved regime. Finally in Sec. 5 we conclude the paper.

2. CAPACITY LIMITS

A basic optical communication link can be characterized by the average received optical power P, bandwidth B and the central frequency f_c. In the photon-starved regime, which we primarily consider in this work, it is convenient to characterize the various power parameters of the link in terms of the average numbers of photons. In particular the average received optical power can be expressed as an average number of photons per time slot $n_a = P/(Bhf_c)$, where h is the Planck's constant. We will use this convention for all power-like quantities in this paper.

In a conventional approach communication links obey the Shannon-Hartley bound.[11] For a link characterized by an average received optical power n_a and noise power n_b, the bound reads

$$C_{\mathrm{SH}} = \log_2 \left(1 + \frac{n_a}{1+n_b}\right). \tag{1}$$

There are two assumptions made in order to derive this bound. The first is that the signal undergoes an additive Gaussian noise in which the electromagnetic field amplitude is shifted by a complex Gaussian random variable β with zero mean and variance $\langle|\beta|^2\rangle = n_b$. A second assumption is that one uses a coherent receiver, typically consisting from a single homodyne or double homodyne detection which allows one to select a particular mode in which signal would be observed.

Crucially, the capacity in Eq. 1 depends solely on the signal-to-noise (SNR) ratio. For very weak signals $C_{\mathrm{SH}} \sim n_a$, and the capacity becomes correspondingly small. It is therefore convenient to use instead the photon information efficiency (PIE), defined as the ratio of the actual capacity to the average received power $\mathrm{PIE} = C/n_a$, in order to compare the performance of different communication strategies in the weak power regime. In this limit, $n_a \to 0$, the PIE corresponding to the double homodyne detection Shannon-Hartley bound is equal to $\mathrm{PIE}_{\mathrm{SH}} = \log_2 e/(1+n_b)$ and single homodyne detection can increase this value by a factor of 2. Note that the asymptotic PIE value is finite even for the vanishing noise power $n_b = 0$. This is a consequence of the fact that homodyne and double homodyne detection are inherently noisy and limited by the shot noise originating from fluctuations of the local oscillator field.[11]

On the other hand, at the fundamental level, the optical communication link is limited only by the laws of quantum mechanics. This means that by using, for example, more exotic receivers, it may be possible to go beyond the Shannon-Hartley bound. This is indeed the case, and the ultimate quantum-mechanical bound on the communication rate of a noisy power-limited optical link reads[12]

$$C_{\mathrm{GH}} = g(n_a + n_b) - g(n_b), \tag{2}$$

where $g(x) = (x+1)\log_2(x+1) - x\log_2 x$ and is known as the Gordon-Holevo bound. Note that the signal and noise powers enter the above expression Eq. 2 separately, not just through the SNR like in Eq. 1. Importantly, the bound,in Eq. 2, similarly as the previous one Eq. 1, is derived for the additive Gaussian noise model and a narrowband, i.e. single mode, channel. The asymptotic value of the PIE for the Gordon-Holevo capacity is equal to $\mathrm{PIE} = \log_2\left(1 + n_b^{-1}\right)$,[13,14] which is much higher than the corresponding Shannon-Hartley bound. In particular, for the noiseless case, $n_b = 0$, the Gordon-Holevo PIE diverges, whereas the Shannon-Hartley one

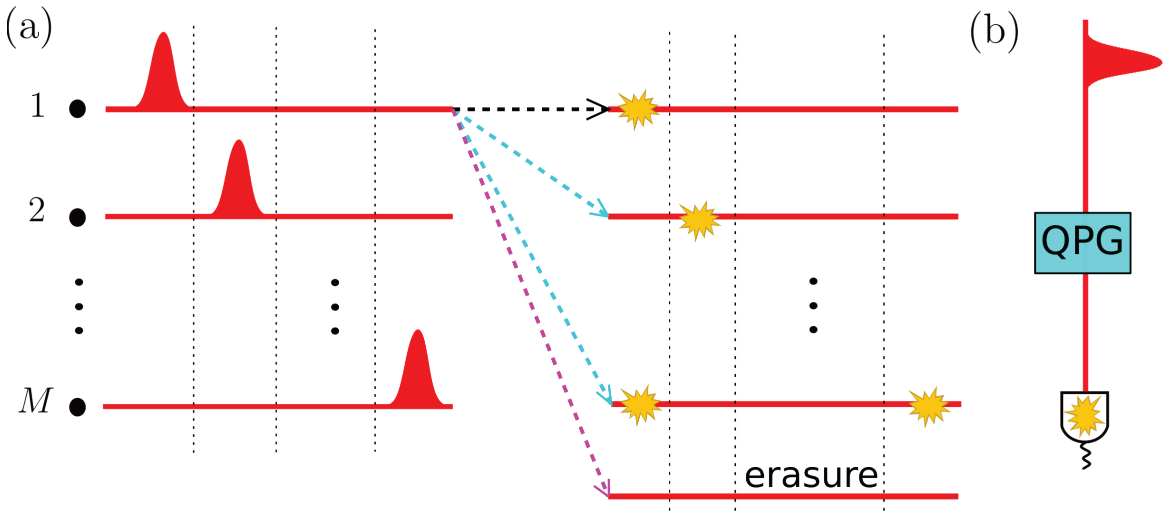

Figure 1. (a) A scheme of PPM format. Different input symbols are encoded in a position of a coherent light pulse within a sequence of M time bins. Time-resolved direct detection performed on the signal in the noiseless case results in detector firing only in the presence of light pulse (black arrow) whereas in the presence of noise it may click also at different positions or multiple time slots (blue arrows). An additional erasure event, indicated by the purple arrow, occurs when no clicks are observed. (b) Mode-selective detection. An impinging noisy light pulse travels through QPG which chooses the signal mode, which is then measured by the detector.

remains finite. This indicates that quantum-enhanced strategies may offer a large enhancement in information transmission rates for weak signal powers.

It is known that the Gordon-Holevo bound can be attained in principle,[15,16] however, from a practical point of view, this is not always the case. This is because the bound assumes an optimal quantum measurement, which may be performed coherently and simultaneously on a large number of time slots. In most realistic scenarios such a device is impossible with current technology. Two notable exceptions are the strong and weak signal regimes. In the former, one can easily show that the bound can be attained, up to 1 nat = 1.44 bits difference, using double homodyne detection, whereas in the latter it is in principle possible to saturate it with generalized on-off keying modulation (OOK) and photon number resolving detection.[17] Since PPM can be viewed as a restricted version of OOK in which only particular sequences of OOK symbols are used, it is worth looking if it also attains the bound.

3. NOISY PPM LINK

In a conventional PPM link, presented schematically in Fig. 1, information is encoded in a series of M time bins from which only one is occupied by a coherent light pulse carrying an average number of photons $n_f = Mn_a$, where n_a is the average received power. A time-resolved direct detection is then performed in each time slot. In the ideal noiseless scenario, one observes detector firing only in the presence of the light pulse, otherwise the detector remains silent. It is therefore possible to unambiguously find the location of the signal pulse in the PPM frame by just inspecting the slots where the detector clicks occurred and easily decode the transmitted symbol. The cases in which the detector did not click, even though there was a light pulse impinging on it, result in an additional erasure event, which causes the transmission to be imperfect. This principle of operation works irrespectively of the power registered at the receiver side, making it particularly useful for weak signals, typical for deep-space communication. Importantly, such simple decoding strategy allows to attain divergent PIE in the weak signal regime, characteristic for the Gordon-Holevo quantum bound.[6,7]

In practical scenarios, however, the signal is always accompanied by some level of noise. Crucially, the presence of noise drastically changes the picture described above. This is because in such case the time bins that previously were empty carry a nonzero amount of photons. There are two consequences of this fact. The first one is that there are multiple click events in which it is not straightforward which PPM symbol should be assigned to the measured sequence of clicks. The second consequence is that even when the detector clicked only once in a given PPM frame one cannot unambiguously decode the corresponding PPM symbol, since the click may be caused by noisy photons in one of the empty time bins. These facts lead to a significantly worse performance of a conventional noisy PPM link since the corresponding capacity scales proportionally to a square of the received average power as compared with a linear scaling, typical for radio communication.[8] This means that for very large distances, where the average received power becomes small, conventional PPM link is not a viable method of communication. In order to save the performance of PPM one needs to use soft-decoding techniques in which information is decoded from the full statistics of registered events.[18] One may show that in such a case the PPM optical link may still out-compete traditional radio communication and offer a considerable advantage in the communication rate.[9,10] Unfortunately, the resulting PIE is still below the Gordon-Holevo bound.[19]

In our model we assume a more general signal detection method utilizing a photon number resolving (PNR) receiver. As opposed to direct detection, in which one can learn only about the presence or not of photons in a particular slot, the PNR receiver allows one to measure the exact number of photons in each time bin. PNR detectors have seen considerable progress in recent years and can operate with very high efficiency[20–22] at the cost of cryogenic cooling requirement. Standard PNR detection, similarly as direct detection, integrates the noise present in all detected orthogonal light modes which results in a Poissonian noise model. This can be circumvented by employing techniques such as quantum pulse gating[23] allowing one to reject all other temporally overlapping orthogonal modes except the selected one. As a consequence, the noise can be represented by the standard additive Gaussian noise model[19] with a significantly reduced average power. One is therefore able to perform a single mode selective PNR detection, which facilitates comparison with the Gordon-Holevo bound. Under the above assumptions the probability of measuring k photons in a PPM signal is equal to[24]

$$p_0(k) = \frac{n_b^k}{(n_b+1)^{k+1}}, \quad p_1(k) = \frac{n_b^k}{(n_b+1)^{k+1}} \exp\left(-\frac{n_f}{n_b+1}\right) L_k\left(-\frac{n_f}{n_b(n_b+1)},\right) \tag{3}$$

for empty time bin and the one occupied by the light pulse respectively, and where $L_k(x)$ denote Laguerre polynomials. Note that for the noiseless case, $n_b = 0$, distribution for the empty slot is nonzero only for $k = 0$, meaning that photocounts are observed only when the actual light pulse is present, as already mentioned above.

4. PHOTON STARVED REGIME BEHAVIOR

The maximum attainable information transmission rate for any given transmission protocol is given by the Shannon mutual information I.[25] For the considered case of noisy PPM with mode selective PNR receiver this quantity can be expressed using probabilities in Eq. 3, however, the resulting formula is quite complicated. Instead, one may use a lower bound[5]

$$I_{\mathrm{PPM}} \geq \frac{1}{M} D(p_1 \| \bar{p}), \qquad \bar{p} = \left(1 - \frac{1}{M}\right) p_0 + \frac{1}{M} p_1. \tag{4}$$

where $D(p\|q) = \sum_k p(k) \log_2[p(k)/q(k)]$ is the Kullback-Leibler divergence between distributions p and q and $\bar{p}$ denotes the observed average probability distribution of photocounts. Importantly, for large PPM orders M the above inequality becomes tight, so one can analyze just the simple expression on the left-hand side. Using now the concavity of the logarithm and the fact that $n_f = M n_a$, it is straightforward to show that

$$\mathrm{PIE}_{\mathrm{PPM}} \geq \log_2\left(1 + \frac{1}{n_b}\right) + O\left(\frac{1}{n_f} \log_2 \frac{n_a}{n_f}\right). \tag{5}$$

where $O(x)$ denotes that all other terms except the first one are of the order of x or smaller. Crucially, the first term in the above expression coincides with the Gordon-Holevo PIE bound and the second one vanishes for large n_f. Fig. 2(a) presents PIE optimized over the average power per frame n_f. It can be seen that the exact PIE converges to the asymptotic value, although it does so very slowly. A similar behavior was observed for the generalized OOK.[17]

The above discussion shows that PPM with mode-selective PNR receiver can attain the ultimate quantum bound. However, in the derivation of this result we assumed $n_f \to \infty$, which is true only for the asymptotic

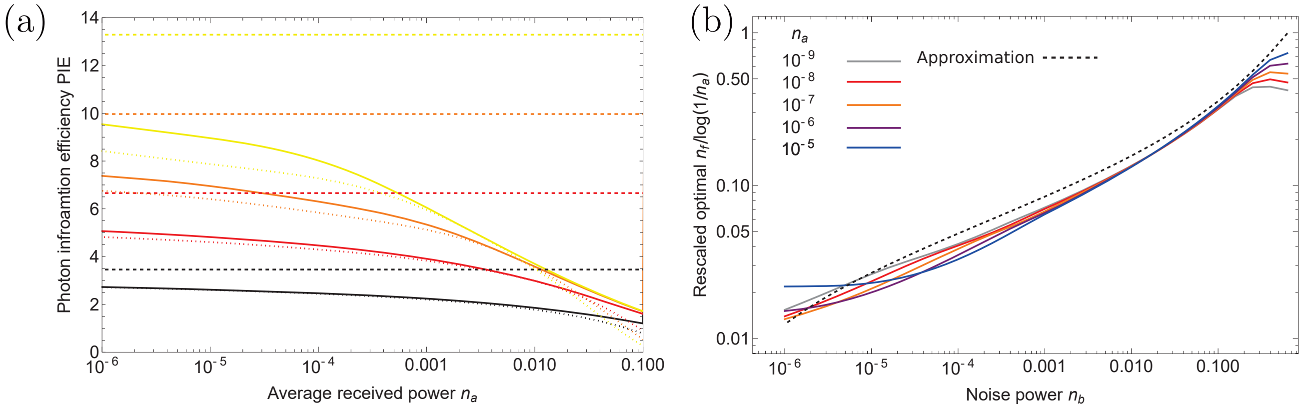

Figure 2. (a) Optimal PIE as a function of the average received optical power n_a for different strengths of the noise: $n_b = 10^{-1}$ - black, $n_b = 10^{-2}$ - red, $n_b = 10^{-3}$ - orange, $n_b = 10^{-4}$ - yellow. Solid curves represent results of optimization of the mutual information, dotted ones are obtained assuming approximate value given in Eq. 6 and dashed lines show asymptotic values from Eq. 5. (b) Optimal value of the received optical power per PPM frame n_f rescaled to the logarithm of the average received optical power $\log \frac{1}{n_a}$ as a function of noise power n_b. Black dashed curve represents the approximate expression Eq. 6.

case of vanishing average received optical power $n_a \to 0$. From a practical perspective, it would be interesting to know what happens for finite values of n_a and in particular what is the optimal power of the PPM light pulse n_f. Fig. 2(b) presents the optimal value of n_f found numerically for the exact PPM mutual information rescaled by the logarithm of the average received optical power. It is seen that in the range of noise power $10^{-5} \leq n_b \leq 10^{-1}$ the curves obtained for different n_a almost coincide, meaning that the average number of photos per frame is practically proportional to $\sim \log \frac{1}{n_a}$ and otherwise depends only on n_b. We found that a good rule of thumb for approximating the optimal value of n_f is given by

$$n_f \approx \frac{\log \frac{1}{n_a}}{\log\left(1 + \frac{1}{n_b}\right)} + A \log \frac{1}{n_a}, \tag{6}$$

where A depends very weakly on n_b and in the range of relevant noise powers can be taken as a constant. We numerically found that $A \approx -0.06$. It is seen in Fig. 2(b) that the above approximation fits the numerically obtained values of n_f pretty well. Similarly, in Fig. 6(a) one can see that PIE calculated with this approximation also agrees with the exact optimal values.

5. CONCLUSIONS

In conclusion, we showed that in the photon-starved regime PPM, when employed together with PNR detection, can attain the ultimate Gordon-Holevo limit on the capacity of a noisy optical communication link. An important assumption is the existence of a way in which noise in the field modes orthogonal to the signal mode can be rejected. Such a procedure is permitted with the help of quantum pulse gating. We showed that the optimal optical power of the PPM light pulse is proportional to the logarithm of the average received optical power and gave a rule of thumb for its behavior. It seems plausible that further developments in the field of PNR receivers and quantum pulse gating may not only open the way to attain the quantum capacity bound but also reduce the noise significantly, further improving the attainable transmission rates.

Importantly, we obtained that for extremely weak signals it is necessary to use light pulses characterized by optical power exceeding the average optical power by several orders of magnitude. This presents potential issues from a practical point of view, as the power budget on a deep-space mission is a considerable constraint. There are, however, some proposals how to bypass this issue by employing BPSK Hadamard words modulation and various collective receiver designs[26,27] that transform BPSK sequences into PPM words, which can then be measured with PNR detection.

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REFERENCES

[1] Tolker-Nielsen, T. and Oppenhauser, G., "In-orbit test result of an operational optical intersatellite link between ARTEMIS and SPOT4, SILEX," in [*Free-Space Laser Communication Technologies XIV*], Mecherle, G. S., ed., **4635**, 1 – 15, International Society for Optics and Photonics, SPIE (2002).

[2] Araki, K., Arimoto, Y., Shikatani, M., Toyoda, M., Toyoshima, M., Takahashi, T., Kanda, S., and Shiratama, K., "Performance evaluation of laser communication equipment onboard the ETS-VI satellite," in [*Free-Space Laser Communication Technologies VIII*], Mecherle, G. S., ed., **2699**, 52 – 59, International Society for Optics and Photonics, SPIE (1996).

[3] Boroson, D. M., Robinson, B. S., Murphy, D. V., Burianek, D. A., Khatri, F., Kovalik, J. M., Sodnik, Z., and Cornwell, D. M., "Overview and results of the Lunar Laser Communication Demonstration," in [*Free-Space Laser Communication and Atmospheric Propagation XXVI*], Hemmati, H. and Boroson, D. M., eds., **8971**, 89710S, International Society for Optics and Photonics, SPIE (2014).

[4] Grecious, T., "Psyche Overview." NASA, 23 December 2021 https://www.nasa.gov/mission_pages/psyche/overview/index.html. (Accessed: 22 August 2022).

[5] Hemmati, H., [*Deep-Space Optical Communication*], John Wiley & Sons, Inc. (2005).

[6] Kochman, Y., Wang, L., and Wornell, G., "Toward photon-efficient key distribution over optical channels," *Information Theory, IEEE Transactions on* **60**, 4958–4972 (Aug 2014).

[7] Jarzyna, M., Kuszaj, P., and Banaszek, K., "Incoherent on-off keying with classical and non-classical light," *Opt. Express* **23**(3), 3170–3175 (2015).

[8] Moision, B. and Farr, W., "Range dependence of the optical communications channel," *IPN Prog. Rep.* **42-199**, 1–10 (2014).

[9] Jarzyna, M., Zwoliński, W., Jachura, M., and Banaszek, K., "Optimizing deep-space optical communication under power constraints," in [*Proc. SPIE 10524, Free-Space Laser Communication and Atmospheric Propagation XXX*], 105240A (2018).

[10] Zwoliński, W., Jarzyna, M., and Banaszek, K., "Range dependence of an optical pulse position modulation link in the presence of background noise," *Opt. Express* **26**, 25827–25838 (2018).

[11] Banaszek, K., Kunz, L., Jachura, M., and Jarzyna, M., "Quantum limits in optical communications," *Journal of Lightwave Technology* **38**(10), 2741–2754 (2020).

[12] Giovannetti, V., García-Patrón, R., Cerf, N. J., and Holevo, A. S., "Ultimate classical communication rates of quantum optical channels," *Nature Photon.* **8**, 796–800 (2014).

[13] Jarzyna, M., "Classical capacity per unit cost for quantum channels," *Phys. Rev. A* **96**(3), 032340 (2017).

[14] Ding, D., Pavlichin, D. S., and Wilde, M. M., "Quantum channel capacities per unit cost," *IEEE Transactions on Information Theory* **65**(1), 418–435 (2019).

[15] Schumacher, B. and Westmoreland, M. D., "Sending classical information via noisy quantum channels," *Phys. Rev. A* **56**, 131–138 (Jul 1997).

[16] Holevo, A., "The capacity of the quantum channel with general signal states," *IEEE Transactions on Information Theory* **44**(1), 269–273 (1998).

[17] Jarzyna, M., "Attaining classical capacity per unit cost of noisy bosonic gaussian channels," *Phys. Rev. A* **104**, 022605 (2021).

[18] Kitayama, K., [*Optical Code Division Multiple Access: A Practical Perspective*], Cambridge University Press (2014).

[19] Banaszek, K., Kunz, L., Jarzyna, M., and Jachura, M., "Approaching the ultimate capacity limit in deep-space optical communication," in [*Free-Space Laser Communications XXXI*], Hemmati, H. and Boroson, D. M., eds., **10910**, 109100A, International Society for Optics and Photonics, SPIE (2019).

[20] Fukuda, D., Fuiji, G., Numata, T., Amemiya, K., Yoshizawa, A., Tsuchida, H., Fujino, H., Ishii, H., Itatani, T., Inoue, S., and Zama, T., "Titanium-based transition-edge photon number resolving detector with 98% detection efficiency with index-matched small-gap fiber coupling," *Opt. Express* **19**, 870–875 (2011).

[21] Gerrits, T., Calkins, B., Tomlin, N., Lita, A. E., Migdall, A., Mirin, R., and Nam, S. W., "Extending single-photon optimized superconducting transition edge sensors beyond the single-photon counting regime," *Opt. Express* **20**, 23798–23810 (2012).

[22] Harder, G., Bartley, T. J., Lita, A. E., Nam, S. W., Gerrits, T., and Silberhorn, C., "Single-mode parametric-down-conversion states with 50 photons as a source for mesoscopic quantum optics," *Phys. Rev. Lett.* **116**, 143601 (2016).

[23] Eckstein, A., Brecht, B., and Silberhorn, C., "A quantum pulse gate based on spectrally engineered sum frequency generation," *Optics Express* **19**(15), 13770 (2011).

[24] Arecchi, F., Berne, A., Sona, A., and Burlamacchi, P., "Photocount distributions and field statistics," *IEEE Journal of Quantum Electronics* **2**(9), 341–350 (1966).

[25] Cover, T. and Thomas, J. A., [*Elements of Information Theory*], John Wiley Sons (2006).

[26] Guha, S., "Structured optical receivers to attain superadditive capacity and the Holevo limit," *Phys. Rev. Lett.* **106**(24), 240502 (2011).

[27] Jachura, M. and Banaszek, K., "Structured optical receivers for efficient deep-space communication," in [*IEEE International Conference on Space Optical Systems and Applications (ICSOS)*], (2017).