

A multilateral fair negotiation model solved by Hungarian solution

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ABSTRACT

Multi-agent was introduced to study the issue of multilateral and multi-attribute fair negotiation in e-commerce. In this paper, a mathematical model is established through the Hungarian solution to solve the problem. Taking into account the multi-attribute and multilateral conditions and fairness in the negotiation, the real negotiation information is simulated in the form of randomly generated data, and the evaluation profit system and bilateral negotiation model are established to solve the problem and then the profit matrix can be obtained. Analogy 0-1 assignment problem is solved by Hungarian solution. Through numerical experiments and analysis, it indicates that this algorithm can obtain the overall maximum profit value that the system can achieve, and make the number of matching people as large as possible to achieve better negotiation results.

Keywords: E-commerce, 0-1 assignment, Hungarian solution, multilateral fair negotiation

1. INTRODUCTION

As a new business operation mode, e-commerce has great practical value. Negotiation is not only a common method to solve problems in human society, but also a key link in e-commerce. In e-commerce, negotiations in the form of “multiple to multiple” (multiple sellers and multiple buyers) are more common. At home and abroad, automatic negotiation in e-commerce environment has been studied in different aspects and depths, and many effective methods have been proposed. Apart from these, there has also been automatic negotiation systems that can support online negotiations. For instance, Faratin et al. proposed several structured negotiation algorithms based on interaction between participants^{1,2}. These negotiation systems can handle bilateral and multi-attribute negotiation through interaction between agents. Park and Yang proposed a multilateral efficient negotiation system³ in pervasive computing environment, and used sorting method to match, achieving good results. Mandeep Mittal et al. proposed a bilateral optimized model for negotiation between buyer and seller through a mediator agent to negotiate on the issue price and the quantity for multi-item⁴. Domestically, Shangwen Xing studied e-commerce automatic negotiation by using Agent negotiation theory and game theory⁵. And Chongping Chen proposed automatic negotiation strategy and utility function⁶ based on time. Liu Jinpeng modeled the online business market and designed an adaptive compromise negotiation mechanism in a dynamic environment⁷. In terms of improving the efficiency of the negotiation model, Gao⁸ combined the Agent matching of e-commerce automatic negotiation with the process of artificial bee colony algorithm, Tang⁹ introduced a metric that is able to evaluate the efficiency of the negotiation process, proposing a novel meta-strategy, Cao et al.¹⁰ constructed a mass customization operation model in an e-commerce environment. Practice has proved that the combination strategy is used to guide the agent to negotiate bids, which is more conducive to improving the negotiation efficiency. In terms of transaction profits, the average matching obtained is relatively high by the existing methods, but the total profit of the whole system is not high enough, and the number of pairings is also not enough³. Based on the second-hand car trading system, we construct a multilateral multi-attribute fair negotiation model on how to maximize the overall profit and improve the matching number, and use the Hungarian solution to solve it.

2. PREPARATORY KNOWLEDGE

2.1 Multilateral multi-attribute fair negotiation

Multilateral and multi-attribute negotiation has always been an important field of e-commerce research. Multilateral refers to the negotiation between multiple buyers and multiple sellers, which has more practical value than bilateral negotiation in reality. Multi-attribute negotiation means that the two sides weigh different attributes in the negotiation scheme under

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the condition that they have a complete understanding of preferences, so that the two sides can achieve a win-win situation. In the actual production and life, multilateral multi-attribute negotiation is very common.

The negotiation system of this paper adopts the conceptual mechanism of confidentiality to ensure fair negotiations. The Intermediary agent received things from buyers and sellers, but buyers and sellers kept secret in their peers. For the entire negotiation system, the intermediary agent can find a mutually beneficial agreement, and select a suitable threshold according to the characteristics of the system, so that the intermediary does not bias any participant, thus ensuring the feasibility and fairness of the negotiation.

2.2. 0-1 assignment problem

2.2.1 *0-1 assignment issues raised.* In this question, N Individuals are responsible for N tasks. Each person has different efficiency for different tasks. One person can only complete one task and one task can only be completed by one person. The purpose is to make the total efficiency of task completion as high as possible.

The 0-1 assignment problem is similar to the multilateral negotiation problem to some extent. N individuals just assume N tasks, corresponding to that one buyer can only complete the transaction with one seller. Each person has different efficiency for different tasks. One person can only complete one task, and one task can only be completed by one person. The purpose of this study is to make the total efficiency of task completion as high as possible, and then extend to make the success rate and profit of the transaction as large as possible.

2.2.2 *The general form of 0-1 assignment problem.* Let n resources (people or machines, etc.) A_1, A_2, \dots, A_n , assign to do n things B_1, B_2, \dots, B_n , require that each thing is completed with only one resource, and the completed resources will not be used by other things. It is known that the efficiency of A_i to do B_j is c_{ij} . Making the overall efficiency best through reasonable assignment is the problem to be solved by 0-1 assignment. $(c_{ij})_{n \times n}$ is called efficiency matrix. the mathematical model of the problem is

$$\left\{ \begin{array}{l} \min \quad z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij} \\ \sum_{j=1}^n x_{ij} = 1 \quad (i = 1, 2, \dots, n), \\ s.t. \quad \sum_{i=1}^n x_{ij} = 1 \quad (j = 1, 2, \dots, n), \\ \quad \quad x_{ij} = 0 \text{ or } 1. \end{array} \right. \quad (1)$$

2.3 Hungarian solution

2.3.1 *Related theorems and concepts of the Hungarian solution.* In 1955, Kuhn proposed a new assignment problem algorithm, called Hungarian algorithm, by using the independent zero element theorem in matrices proposed by Hungarian mathematician¹¹. The Hungarian solution has two related theorems:

Theorem 2.1 Suppose $C=(c_{ij})_{n \times n}$ is an efficiency matrix. If n $C=c_{ij}$ corresponding to n 1 of feasible solution $X^*=x_{ij}$ is 0, X^* is the optimal solution. (If all c_{ij} are 0, the final cost is 0, so X^* is the optimal).

Theorem 2.2 For the assignment problem, the new assignment problem obtained by subtracting (or adding) one same number from any row (or column) of the efficiency matrix is the same as the original problem.

Theorem 2.1 and Theorem 2.2 are the principles of the Hungarian solution. These principles convert some elements of the efficiency matrix into 0 by certain operations. If there is a set of 0 elements, it satisfies:

- The number of 0 is equal to the order of the efficiency matrix (i.e. the number of tasks).
- Any two 0 of this set of 0 elements are in different rows and columns of the matrix.

Then this group of zero corresponding allocation is the optimal solution, we call this group of 0 elements as independent zero elements. The definition of the independent zero elements is given as follows:

Definition 2.1 The independent zero elements are the 0 elements in matrix C which are located in neither the same row nor the same column. The minimum number of lines required to cross out all 0 elements in matrix C is equal to the number of independent zero elements in matrix C, that is, the maximum number of 0 selected of different rows and columns.

2.3.2 *The steps of Hungarian solution.* The steps of the Hungarian solution are as follows:

- Line specification: subtract the smallest element in each row from all elements of each row.
- Column specification: do the same for each column so that at least one zero element appears in each column of the coefficient matrix.
- Trial assignment: determine independent 0 elements, if the number of independent 0 elements is equal to the order of coefficient matrix n (trial assignment is successful), the optimal solution can be obtained; if the number of independent 0 elements is less than the order of coefficient matrix n, the optimal solution fail to be obtained (trial assignment fails).
- Draw lines to cover 0 elements: cover all 0 elements by drawing the least number of horizontal and vertical lines, in which the number of lines is the number of independent 0 elements.
- Update matrix: find the smallest number among the unmarked numbers, subtract the minimum number from all the unmarked numbers, and add the minimum number from the numbers drawn by two lines.
- Repeat steps “trial assignment” through “update matrix” until the conditions for successful trial assignment are met.

3. MULTILATERAL MULTI-ATTRIBUTE FAIR NEGOTIATION MODEL

3.1 Scene description

Because the second-hand car trading system has good consumption tendency attribute, this paper studies the multilateral fair consultation model under this background. In this system, more than one seller will be selling the same model of car, thus will negotiating with different customers according to the different attribute values of each vehicle - price, mileage, year of production, warranty date. Both the buyer and the seller have their own emphasis on different attributes, namely weight values, with prices, years of production, mileage, and warranty options weighing respectively 0.5, 0.2, 0.1, and 0.2.

In Table 1, P_{req} and M_{req} are the seller’s request values for the price and the mileage (the maximum that the participant wants in the negotiation). And in Table 2, P_{aw} and M_{req} are the buyer’s allowable values for price and mileage (the maximum that the participant can bargain for). In this paper, we want to explore how to negotiate to maximize the profit of the whole system and match successfully as many as possible. The negotiation information range of 50 buyers and 50 sellers simulated by computers are shown in Tables 1 and 2.

Table 1. The scope of negotiation information generation of the seller.

Attribute	Request value	Allowable value	Weight
Price (USD)	$22000 \leq x \leq 2000$	$[P_{req}-6000, P_{req}-3000]$	0.5
Year of production (year)	[1997, 2000]	[2001, 2004]	0.2
Mileage (miles)	$100 \leq M_{req} \leq 150$	$[M_{req}-70, M_{req}-10]$	0.1
Warranty period (month)	[2, 6]	[12, 36]	0.2

Table 2. The scope of negotiation information generation of the seller.

Attribute	Request value	Allowable value	Weight
Price (USD)	$[P_{aw}-6000, P_{aw}-3000]$	$22000 \leq x \leq 32000$	0.5
Year of production (year)	[2001, 2004]	[1997, 2000]	0.2
Mileage (miles)	$[M_{req}-70, M_{req}-10]$	$100 \leq M_{req} \leq 150$	0.1
Warranty period (month)	[12, 36]	[2, 6]	0.2

3.2 Establishment of the model

3.2.1 *Evaluation of profits.* In this paper, the participants' profits are evaluated using the multi-attribute utility theory (MAUT)¹². The utility function, which can be thought of as a buyer's or buyer's profit, is represented by the weight value w_i of the attribute and the evaluation function E_{x_i} . A participant's profit can be expressed as the following equations:

$$Profits(x_i) = \sum_{i=1}^n w_i \cdot E(x_i) \quad (2)$$

$$\sum_{i=1}^n w_i = 1 \quad (3)$$

$$E(x_i) = \frac{x_i - allowable_value_i}{request_value_i - allowable_value_i} \quad (4)$$

Among these, n represents the serial number of the attribute, x_i represents the variable value of attribute, and w_i means the weight value of the i -th attribute. The $allow_value_i$ and $request_value_i$ are respectively the allow value and request value of the i -th attribute. Therefore, when x_i changes between the $allow_value_i$ and the $request_value_i$, the corresponding E_{x_i} range from 0 to 1. E_{x_i} now represents the satisfaction of attribute x_i , which is set to 0 if the value of E_{x_i} is less than 0. And if the value of E_{x_i} is greater than 1, then it should be set to 1.

$$CNR_{i\ lower} \leq x_i \leq CNR_{i\ upper} \quad (5)$$

$$CNR_i = \{R_{i\ buyer}\} \cap \{R_{i\ seller}\} \quad (i = 1, \dots, n) \quad (6)$$

The $R_{i\ buyer}$ in equation (6) represents the scope of the buyer's negotiation, $R_{i\ seller}$ in equation (6) represents the seller's negotiation scope, and CNR_i represents the intersection of the buyer's and seller's negotiating scope for the i -th attribute.

In order to improve the efficiency of the algorithm, we firstly pretreat the data. We pick the group impossible to match, that is, the two attribute ranges intersect empty, set the corresponding profit matrix P elements to 0, and for the three attribute intersections are non-empty, set the corresponding profit matrix P elements to 0.05 (a constant that is not 0). It's more convenient for calculating. And in the next MATLAB profit matrix calculation program¹³, they will be discussed further.

Since each element of the profit corresponds to a pair of matching optimal profit values, each match is equivalent to a bilateral negotiation. If a pair matches such as the profit $P_{(i,j)}$ corresponding profit is 0, it is not possible to match; if the corresponding profit is not 0, then we take the s -th seller and the t -th buyer for example:

$$Profits^{seller}(x_i) = \sum_{i=1}^n w_i \cdot E(x_i) = 0.5 \times \frac{x_1 - B(1, s)}{A(1, s) - B(1, s)} + 0.2 \times \frac{x_2 - D(1, s)}{C(1, s) - D(1, s)} + 0.1 \times \frac{x_3 - F(1, s)}{E(1, s) - F(1, s)} + 0.2 \times \frac{x_4 - H(1, s)}{G(1, s) - H(1, s)} \quad (7)$$

$$Profits^{buyer}(x_i) = \sum_{i=1}^n w_i \cdot E(x_i) = 0.5 \times \frac{x_1 - B_1(1, t)}{A_1(1, t) - B_1(1, t)} + 0.2 \times \frac{x_2 - D_1(1, t)}{C_1(1, t) - D_1(1, t)} + 0.1 \times \frac{x_3 - F_1(1, s)}{E_1(1, s) - F_1(1, s)} + 0.2 \times \frac{x_4 - H_1(1, s)}{G_1(1, s) - H_1(1, s)} \quad (8)$$

Equations (7) and (8) represents the profit of seller $Profits^{seller}(x_i)$ and the profit of the buyer $Profits^{buyer}(x_i)$. A, B, C, D, E, F, G, H are the request and allowable values of the seller in Table 3 for the four properties in turn. In the same way, $A_1, B_1, C_1, D_1, E_1, F_1, G_1, H_1$ represents the request and allowable values of the buyer's four attributes in turn.

3.2.2 *Establishment of the negotiation model.* This paper takes the attribute of negotiation as the variable, and establishes the bilateral reciprocal negotiation model as follows:

$$\max z = Profits^{buyer}(x_i) + Profits^{seller}(x_i) \quad (9)$$

$$\left| Profits^{buyer}(x_i) - Profits^{seller}(x_i) \right| \leq \delta \quad (10)$$

In equations (9) and (10), z is the best profit value, the sum of the profits of the buyer and seller, n is the quantity of the product attribute, and δ is the threshold for the difference in profits between the buyer and the seller. Considering the characteristics of negotiation, this paper sets an appropriate threshold $\delta=0.01$, which means that the profit bias between a buyer and a seller is not more than 1%.

3.2.3 Establishment of allocation model. Similarly, we can draw an analogy between matching in a system and determining assignment in a 0-1 assignment problem. The objective function of the distribution model is to obtain the maximum profit income of the total system. In this paper, the maximum value of each pair of possible matches can be regarded as a new attribute of both parties, and the optimal value of all possible matches between all buyers and all sellers is formed into a profit matrix. In this matrix, the corresponding value of the impossible match is set to 0. The following allocation model is established:

$$\max w = \sum_{i=1}^m \sum_{j=1}^m c_{ij} x_{ij} \quad (11)$$

$$\sum_{j=1}^m x_{ij} = 1 \quad (i = 1, 2, \dots, m) \quad (12)$$

$$\sum_{i=1}^m x_{ij} = 1 \quad (i = 1, 2, \dots, m) \quad (13)$$

$$x_{ij} = 0 \text{ or } 1 \quad (i = 1, 2, \dots, m; j = 1, 2, \dots, m) \quad (14)$$

In the equations above, w is the total profit value of the system, a maximum of w is required in this paper. The i -th line j -th column element in the matrix C represents the maximum profit value that the i -th seller can achieve by matching the j -th buyer (that is, the z value obtained by the i -th seller and the j -th buyer in the negotiation model), m means the number of buyers and sellers. Decision variables are x_{ij} , the relationship is shown in equation (15):

$$x_{ij} = \begin{cases} 1, & \text{the } i \text{ seller matches the } j \text{ buyer} \\ 0, & \text{the } i \text{ seller fails to match the } j \text{ buyer} \end{cases} \quad (15)$$

Equations (12) and (15) indicate that a seller can match only one buyer at most. Equations (13) and (15) indicate that a buyer can only match one seller at most. If when x_{ij} corresponding c_{ij} value is 0, then the i -th seller and the j -th buyer made a “virtual match”, which means no match. And this distribution model is supposed to match the same number of buyers and sellers. Of course, it’s possible to include “virtual matches”, which will continue to be discussed later in the program.

4. SIMULATION EXPERIMENT

4.1 Experimental results

What we want is the maximum value of equation (9) under the given conditions. According to the function of solving the linear planning problem in MATLAB as the optimal profit value z , i.e. the element value corresponding to the profit matrix $P(s,t)$, which makes up the 50×50 profit matrix. And it is matched by the classic Hungarian solution in the 0-1 assignment problem. Results solved by MATLAB is shown in Table 3.

Table 3. MATLAB running result.

	1	2	3	4	5	6	7	8
Total profit (\$ 10,000)	53.456	59.801	61.688	60.661	56.372	55.675	57.349	56.293
Matching logarithm (pair)	50	49	50	50	50	50	50	50
Average profit (\$10,000)	1.069	1.220	1.234	1.213	1.217	1.114	1.147	1.149
Match time (seconds)	0.517	0.397	0.422	0.482	0.602	0.464	0.481	0.565

Sanghyun Park proposed in 2008 in the multilateral consultative system³ in the distribution, in which the use of profit sorting method spent most of the time of the whole algorithm. Its time complexity is $O(N \log N)$, where N is the number of consultations. The time complexity of the Hungarian solution mentioned in this paper is $O(N^2)$. Although the sorting algorithm is better than the Hungarian algorithm when $N \rightarrow \infty$, the total profit of this paper must be optimal, and the matching logarithm is guaranteed.

4.2 Effects of changes in relevant parameters on experimental results

We set $\delta=0.01$ in this paper, and the experimental results have been obtained. But by changing the threshold $\delta=0.005, 0.02, 0.05$, the results are not remarkably different. Four sets of data were compared, as shown in Table 4.

Table 4. The influence of threshold on data.

δ	0.005	0.01	0.02	0.05
Group one	56.2216	56.2446	56.2923	56.4260
Group two	55.1049	55.1278	55.1715	55.3099
Group three	59.3926	59.4150	59.4564	59.5643
Group four	55.0366	55.0668	55.1268	55.2877

5. CONCLUSION

In this paper, we study the multilateral fair negotiation model based on e-commerce environment, mainly for the second-hand car trading system, establishing the multilateral fair negotiation model and calculating the profit matrix, considering the integrity of the trading system, choosing the Hungarian solution to solve the distribution model. Through numerical experiments and analysis, the maximum total profit and as many matching numbers can be obtained by the method proposed in this paper. Therefore, it has great research value for the actual trading system. Further research may include improving the stability of matching parties, average profit and simplified solution.

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