Optical viewer based on integral method for three-dimensional images

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ABSTRACT

We propose an optical viewer based on the integral method. The viewer composed of two GRIN lens arrays and a diffuser in between can form observable three-dimensional images of objects. The length of the elemental GRIN lenses that constitute the arrays is three quarters of the cycle of the meandering ray path on the input side and one quarter of the cycle on the output side. By substituting the diffuser with an image intensifier as an optical amplifier, we were able to observe 3-D images of objects placed in a dark space without the use of a camera or display equipment. The primitive experimental results proved the viewer produces three-dimensional images that can be observed. We also describe the visual resolution characteristics.

Key words: Optical viewer, GRIN lens, integral method, three-dimensional image, image intensifier.

1. INTRODUCTION

Figures 1 and 2 illustrate the principle behind the basic integral method[1-6] using a lens array during both the pickup and display stages. To produce an integral image, a lens array composed of many convex elemental lenses is positioned immediately in front of the photographic film as a pickup device. The integral image is composed of numerous small elemental images, which are imaged on the film, with their number corresponding to the number of elemental lenses. The film is then developed to obtain a transparent photograph. This transparent photograph as a display device is placed where the film had been and is irradiated from behind by a diffused white light source in the display stage. The light rays passing through the photograph retrace the original routes and then converge at the point where the object had been, forming an auto-stereoscopic 3-D image.

The total number of pixels N_t that the film requires is the product of the number of elemental lenses N_m and the number of pixels in an elemental image N_e:

$$N_t = N_m N_e. \quad (1)$$

The number of elemental lenses N_m determines the upper limits of resolution, which corresponds to the number of pixels for the conventional two-dimensional television. The number of pixels in each elemental image affects the resolution of the 3-D image away from the lens array. The film requires N_e times the number of pixels compared with conventional television. Thus, extremely high resolution is required for both the pickup and display devices.

The problem shown in Fig. 1 is that the reconstructed image is viewed from the opposite direction to that of the image pickup. This yields a pseudoscopic image with reversed depth, and so observers see a convex reproduced image when a concave object is captured. Several ways have been proposed [2,3,6] to avoid pseudoscopic 3-D images. The authors also proposed one approach that uses radial gradient index lenses (GRIN[7] lenses) for the pickup-lens array. A GRIN lens easily produces an erect elemental image of an object located at a distance. As a result, pseudoscopic images with reversed depth are avoided (Fig.2)[8-10]. The lens array at the display stage can also be composed of elemental GRIN lenses.

The optical viewer we propose in which pickup and display stages are unified is composed of two GRIN lens arrays and a diffuser in between. The viewer can form observable 3-D images of objects based on the principle of the integral method. By substituting the diffuser with an image intensifier as an optical amplifier, we were able to form 3-D images of objects placed in a dark space. It could be applied to various purposes. The primitive experimental results

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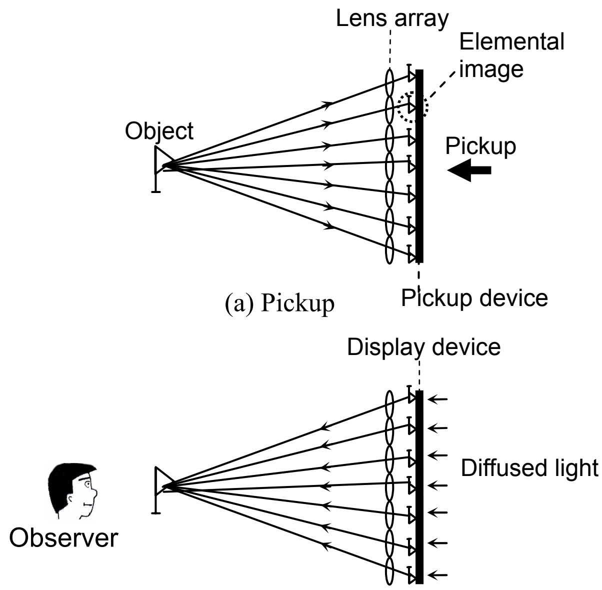
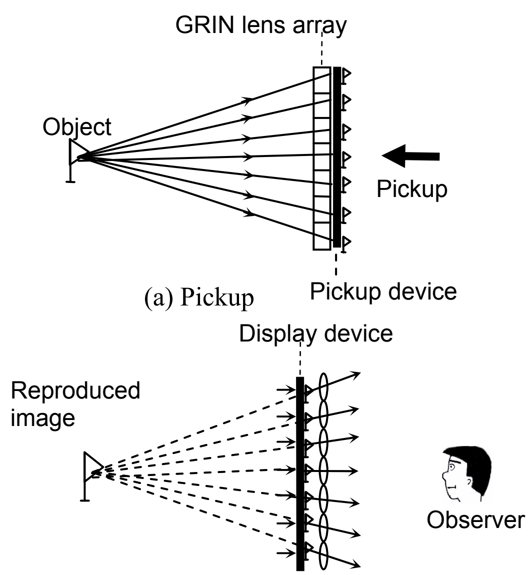


(b) Display (reversed depth)

Fig. 1: Basic principle behind integral method



(b) Display (correct depth)

Fig. 2: Integral method with orthoscopic image

proved the viewer produces 3-D images that can be observed. The visual resolution characteristics are also described in this paper.

2. IMAGE FORMATION USING GRIN LENS ARRAY

The following describes the principle behind the optical viewer with the GRIN lens array. As we can see from Eq. (2), the refractive index of the GRIN lens peaks at the optical axis and decreases farther away from it, which corresponds to square-law characteristics (Fig. 3(a)).

$$n(r) = n_0\left(1 - \frac{1}{2}A_G r^2\right) \tag{2}$$

Here, r is the distance from the optical axis, $n(r)$ is the refractive index in the direction of r, n_o is the refractive index on the optical axis, and $A_G^{1/2}$ is the gradient constant. Light rays from a distant object enter the GRIN lens as near-parallel rays. They then curve in the direction of a higher refractive index, so that the ray path is meandering cyclically, as can be seen from Fig. 3(b). If the GRIN lens length is one cycle of the ray path, L_p, (or integer multiple of one cycle length), the GRIN lens' characteristics are equivalent to afocal optics. The light rays that enter the incident plane will exit at the same angle as the incident angle. This two-dimensional array of the GRIN lenses forms a 3-D image that is the same size as the object[11].

Let us now consider a unit composed of two GRIN lenses and a diffuser in between. The lengths of the GRIN lenses are $3/4 \cdot L_p$ and $1/4 \cdot L_p$. The former can easily form an erect image on the exit plane of the lens for a far object. An erect image is needed to reconstruct a 3-D image with the correct depth[8-10]. If the inverted image is formed by a convex lens instead of the former, the reconstructed 3-D image is pseudoscopic with inverted depth. The array of GRIN lenses also enables optical cross talk between adjacent lenses to be eliminated by applying optical black to the boundaries. The ray matrix of light passing through the former lens is expressed as

$$\begin{pmatrix} r_{1c} \\ r'_{1c} \end{pmatrix} = \begin{pmatrix} 0 & -\dfrac{1}{n_0\sqrt{A_G}} \\ n_0\sqrt{A_G} & 0 \end{pmatrix} \begin{pmatrix} r_{1s} \\ r'_{1s} \end{pmatrix}. \tag{3}$$

Here, r_{1s}, r'_{1s}, r_{1c}, and r'_{1c} are the radial position of incident light, slope of incident light, radial position of output light, and slope of output light, in this order. We placed a diffuser at the exit plane of the lens. The ray matrix of the diffuser is expressed as

$$\begin{pmatrix} r_{dc} \\ r'_{dc} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & D \end{pmatrix} \begin{pmatrix} r_{ds} \\ r'_{ds} \end{pmatrix}. \tag{4}$$

In the same manner as the GRIN lens with $3/4 \cdot L_p$ of the length, r_{ds}, r'_{ds}, r_{dc}, and r'_{dc} represent these factors. Here, we assumed the light rays would diffuse in all directions of the hemisphere, and the D was a real number that would satisfy $-\pi/2 \le D \cdot r'_{ds} \le \pi/2$. Further, the other GRIN lens with $1/4 \cdot L_p$ of the length is attached to the diffuser. This GRIN lens is equivalent to convex lens. Light rays from the image formed on the diffuser will be output through this second GRIN lens as parallel light beams. The ray matrix of the lens is expressed as

$$\begin{pmatrix} r_{2c} \\ r'_{2c} \end{pmatrix} = \begin{pmatrix} 0 & \dfrac{1}{n_0\sqrt{A_G}} \\ -n_0\sqrt{A_G} & 0 \end{pmatrix} \begin{pmatrix} r_{2s} \\ r'_{2s} \end{pmatrix}. \tag{5}$$

In the same manner, r_{2s}, r'_{2s}, r_{2c}, and r'_{2c} represent these factors. The total length of these two GRIN lenses is one cycle of the ray path. The two-dimensional alignment of the elemental units functions like afocal-array optics[11,12], forming a 3-D image.

The GRIN lens with $3/4 \cdot L_p$ of the length, the diffuser, and the GRIN lens with $1/4 \cdot L_p$ of the length are connected in the elemental unit, and we can set $r_{2s} = r_{dc}$, $r'_{2s} = r'_{dc}$, $r_{ds} = r_{1c}$, $r'_{ds} = r'_{1c}$. As a result, the matrix of the elemental unit is expressed as

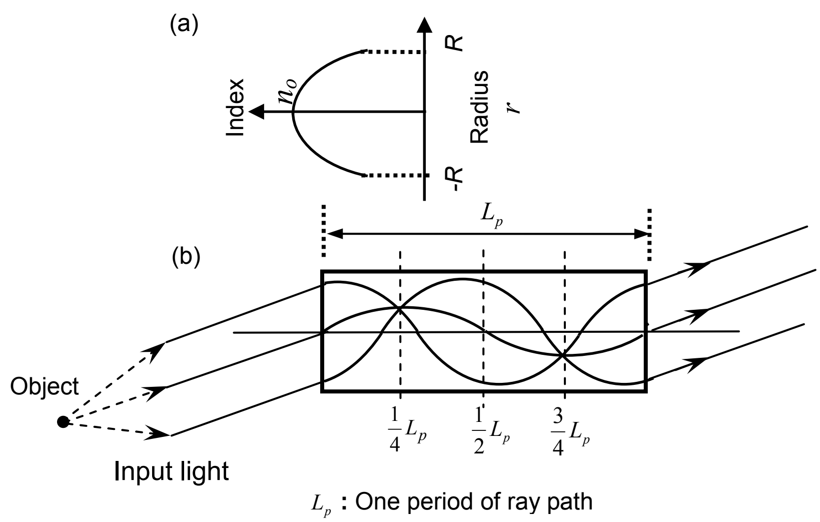


Fig.3: (a) Refractive index of radial GRIN lens.
(b) Light rays through GRIN lens.

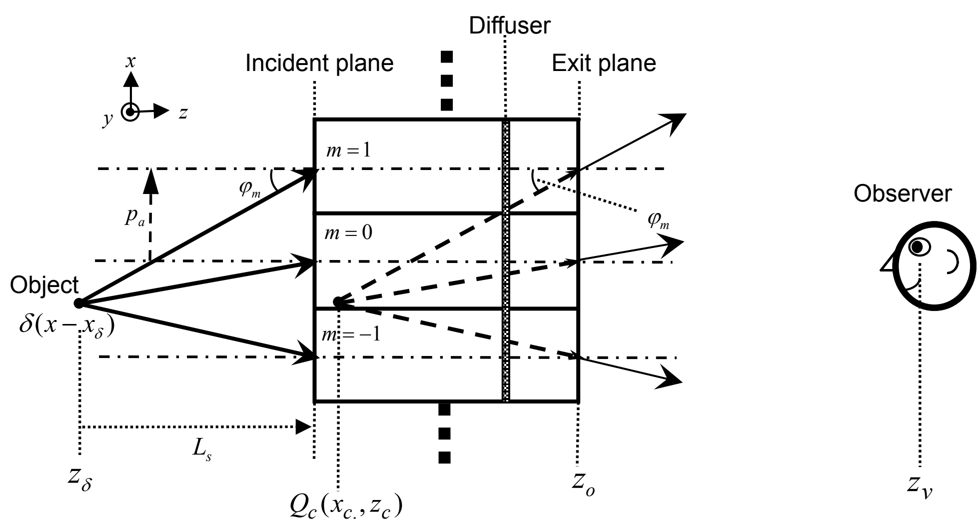


Fig. 4: Geometric view of optical viewer

$$\begin{pmatrix} r_{2c} \\ r'_{2c} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{n_0\sqrt{A_G}} \\ -n_0\sqrt{A_G} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & D \end{pmatrix} \begin{pmatrix} 0 & -\frac{1}{n_0\sqrt{A_G}} \\ n_0\sqrt{A_G} & 0 \end{pmatrix} \begin{pmatrix} r_{1s} \\ r'_{1s} \end{pmatrix} = \begin{pmatrix} D & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} r_{1s} \\ r'_{1s} \end{pmatrix} \quad . \tag{6}$$

Therefore, we obtain: $r_{2c} = D \cdot r_{1s}$. As the D does not have a fixed value, the position where the light rays output the elemental unit on the exit plane is not fixed at one point. However, $r'_{2c} = r'_{1s}$, i.e., the output angle is the same as the incident angle. Here, the incident angle of the m-th elemental unit has been defined as φ_m in Fig. 4. If the diameter of the elemental unit is sufficiently small, the incident angle on the optical axis represents all the angles by which the light rays input the elemental unit, so that we can obtain

$$\frac{mP_a - x_\delta}{L_s} = \frac{mP_a - x_c}{z_o - z_c} = \tan\varphi_m \quad . \tag{7}$$

From the equation, we obtain

$$(z_o - z_c - L_s)mP_a - x_\delta(z_o - z_c) + L_s x_c = 0 \, . \tag{8}$$

Here, P_a is the pitch of the elemental unit. This equation is valid for all elemental units; as a result, $z_c = z_o - L_s, x_c = x_\delta$. Therefore, the light rays exiting the array converge at point $Q_c(x_\delta, z_o - L_s)$. That is, the optical image is formed at this point. This method works just like the integral method. The distance of the 3-D image from the exit plane is the same as the object distance from the incident plane. The size of the image is exactly the same as that of the object. By substituting the diffuser with an image intensifier as an optical amplifier, we are able to form observable 3-D images of objects placed in a dark space.

3. EXPERIMENT FOR FORMING IMAGES

Figure 5 outlines the optical viewer with the image intensifier we used in the experiment. The specifications for the GRIN lens array and the image intensifier are listed in Tables 1 and 2. The GRIN lens of array 1 on the input side is $3/4 \cdot L_p$ of the length, and the one of array 2 on the output side is $1/4 \cdot L_p$ of the length. Objects in this experiment were observed through the optical viewer. Two objects, the letters I and E, were placed in a dark space. Figures 6 (a)-(d) are photographs of the displayed images at the four viewpoints. The objects have an illuminance of about 0.1 lx, and the amplification of the image intensifier was adjusted so that the output phosphor screen had a peak luminous emittance of about 200 lx (lm/m^2). As a result, we could observe 3-D images with visible brightness. The resolution of these images was low as the viewer was not equipped with sufficient number of elemental units. Even so, these photos do show the distinctive positions of the I and E images, indicating that they are presented three-dimensionally. The length of the GRIN lens in this experiment was set at $3/4 \cdot L_p$ of the length on the input side and $1/4 \cdot L_p$ of the length on the output side, but these lengths could be interchanged. The phosphor substance on the exit plane of the image intensifier was P43, which is sensitive to green. The output images looked monochromatic.

4. VISUAL RESOLUTION

An observer watches the optical image formed by the viewer as shown in Fig. 7. The image is located behind the display plane, and is virtual. The light waves from the elemental lenses continue to exit, as emitted from the image, and a portion of the waves enter the observer's pupil. The image on the viewer's retina corresponds to the image formed on the image plane. Therefore, we need to consider the synthesized waves on the image plane to estimate the visual resolution. Since the light waves from the viewer are incoherent, the intensity of the synthesized waves is given by the sum of the squared amplitude of the waves from elemental lenses. Therefore, the MTF of the viewer is given as that of an elemental lens, and can be calculated as the Fourier transform of the squared amplitude of the point spread function[13]. If the elemental lens of the pickup stage and that of the display stage have the same diameter, the MTFs of both stages are the same, i.e., $MTF_L(\alpha)$. Here, α (cycle /radian) is the spatial frequency normalized by object distance L_s.:

$$\alpha = uL_s \tag{9}$$

The u (cycle/m) is the spatial frequency of the objects or the image**.** The overall MTF[14] can then be expressed as

$$MTF_T(\alpha) = [MTF_L(\alpha)]^2 \ , \tag{10}$$

We assumed that the elemental lens was circular, and the MTF was given by

$$MTF_L(\alpha) = \frac{4r_a}{A_c b}\cdot(K_1\cos\varphi - K_2\sin\varphi)\,, \tag{11}$$

$$K_1 = 2[J_1(r_a b)(\frac{\theta}{2}+\frac{\sin 2\theta}{4}) - J_3(r_a b)(\frac{\sin 2\theta}{4}+\frac{\sin 4\theta}{8}) + J_5(r_a b)(\frac{\sin 4\theta}{8}+\frac{\sin 6\theta}{12}) - \cdots]\,, \tag{12}$$

$$K_2 = J_0(r_a b) - 2[J_2(r_a b)(\frac{\sin\theta}{2}+\frac{\sin 3\theta}{6}) - J_4(r_a b)(\frac{\sin 3\theta}{6}+\frac{\sin 5\theta}{10}) + J_6(r_a b)(\frac{\sin 5\theta}{10}+\frac{\sin 7\theta}{14}) - \cdots]\,, \tag{13}$$

$$A_c \equiv r_a^2\pi\,, \tag{14}$$

$$\theta = \cos^{-1}\left(\frac{\lambda u L_s}{2r_a}\right) = \cos^{-1}\left(\frac{\lambda\alpha}{2r_a}\right), \tag{15}$$

$$b = 2\pi u\,, \tag{16}$$

where A_c is the pupil area of the elemental lens, and r_a is the lens' radius.

We used visual spatial frequency v (cycle/radian) to estimate the resolution when an image formed by the viewer was watched by an observer. It is given as a function of spatial frequency u on the image plane and the distance between the observer and the image:

$$v \cong u\cdot|z_v - z_c| = u\cdot|z_v - (z_o - L_s)| = u\cdot|L_{ov} + L_s|\,, \tag{17}$$

$$L_{ov} = z_v - z_o\,, \tag{18}$$

The image should be located in area $z_c \le z_o$. If the virtual object is positioned at the observer's side of the incident plane, $z_c > z_o$ is permitted.

The number of pixels generally used in televisions and other image-display systems is set at a level where the pixel structure cannot be seen by the observer. The same way is applied to avoid the alignment of the elemental lenses from being perceived. These viewing conditions are dependent on the observer's eyesight and the distance from the observer to the image. Here, the observer's eyesight has been assumed to be 20-20, which has a resolution of 1.0 minute in a visual angle. A spatial frequency corresponding to the observer's eyesight, called the visual-limit spatial frequency, is given by

$$v_H = \frac{1}{2}\times\frac{360\times 60}{2\pi} = 1720 \text{ (cycle/radian: cpr).} \tag{19}$$

In addition, the actual resolution deteriorates due to the aberration of the elemental lens. This deterioration occurs in the same way as when GRIN lenses are adopted as elemental lenses. However, MTF was calculated using only the influence of diffraction including focus defects because it was assumed that ideal lenses were used as elemental lenses to obtain results under ideal condition.

Figure 8 plots the MTFs in relation to the viewing spatial frequencies, where the distance to the object is 1.0 m, and the diameters of the lenses correspond to 0.25, 0.5, and 1.0 mm. Viewing distances that give the visual-limit spatial frequency are 0.86, 1.72, and 3.44 m, respectively. As the graph shows, the smaller the diameter of the elemental lens, the smaller the MTF is. Figure 9 plots the MTF responses at the visual-limit spatial frequency (1720 cpr). As previously described, the diameters of the elemental lenses are 0.25, 0.5, and 1.0 mm. The MTF responses decrease as the object distance increases. Because of diffraction, this tendency is greater when the radius of the elemental lens is smaller. When the lens is 1.0 mm in diameter, the MTF is not zero even if the object is placed, sufficiently far away, at a distance of 100 m, thus allowing high-quality images to be observed.

5 . SUMMARY

We demonstrated that the optical viewer based on integral method can form 3-D images. The viewer with the image intensifier can produce especially observable 3-D images of objects placed in a dark space without the need for a camera or display equipment. The viewer forms monochromatic images. We can produce these images in colors, if we use white phosphor as a display plate of the image intensifier and insert color filter arrays on the incident plane of the array1 and the exit plane of the array2. An example of the color filter array is shown in the Fig.10. The diameter of the elemental lens needed to be 1.0 mm to observe distant objects; however, the diameter is permitted to be smaller to observe near objects. This device would be useful for observing 3-D objects in dark space. For example, we could apply it to monitoring the habits of nocturnal animals or viewing disaster spots at night.

Table 1: Specifications for lens arrays

Length of elemental lens	Array 1: 20.25 mm Array 2: 6.75 mm
Diameter of elemental lens	1.085 mm
Diameter of array	25 mm(Active area)
Number of elemental lenses	450(Active area)
Alignment of elemental lenses	Delta

Table 2: Specifications for image intensifier

Luminous gain	Variable: 120–23,000
Diameter of image area	25 mm
Thickness	18.5 mm
Phosphor of display plate	P43

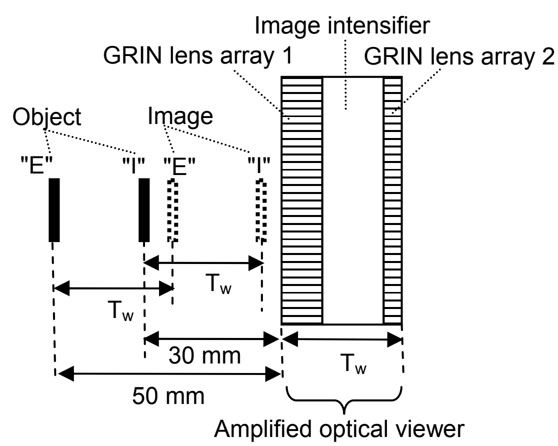


Fig. 5: Experiment using amplified optical viewer (side view). Objects I and E are letters both with 32-point font. I is 30 mm from array 1 and and E is 50 mm from same array. Center of both letters is offset 6 mm horizontally. Both images are separated from two object positions by Tw (45.5 mm), i.e., thickness of amplified optical viewer.

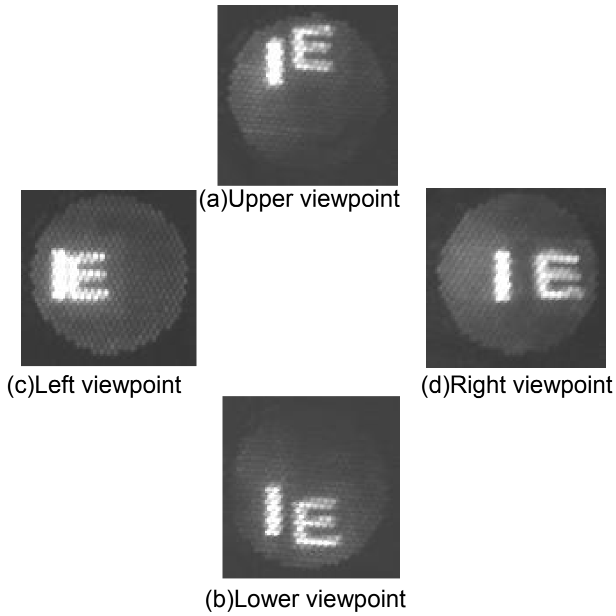


Fig. 6: Photographs of images formed by the optical viewer. Four viewpoints are all away from center axis of exit plane by approx. 9 deg.

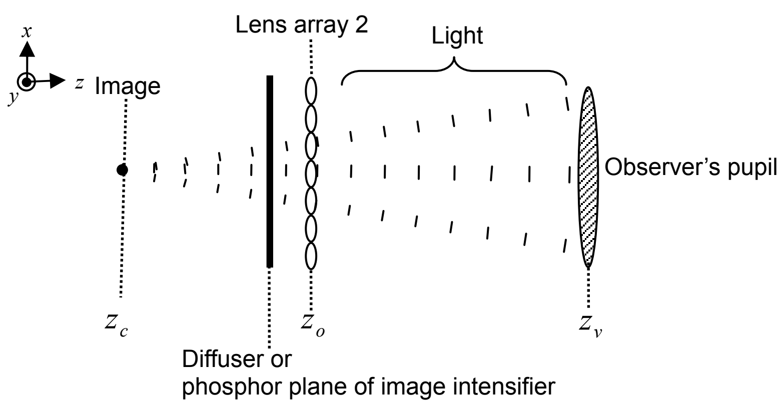


Fig. 7: Reproduced image behind exit plane of optical viewer

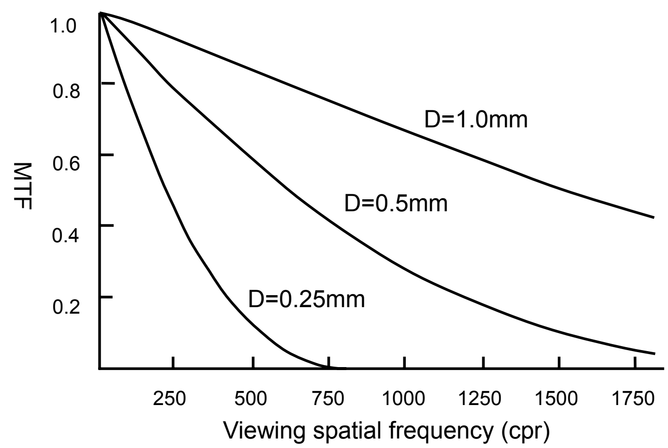


Fig. 8: MTFs corresponding to viewing spatial frequencies

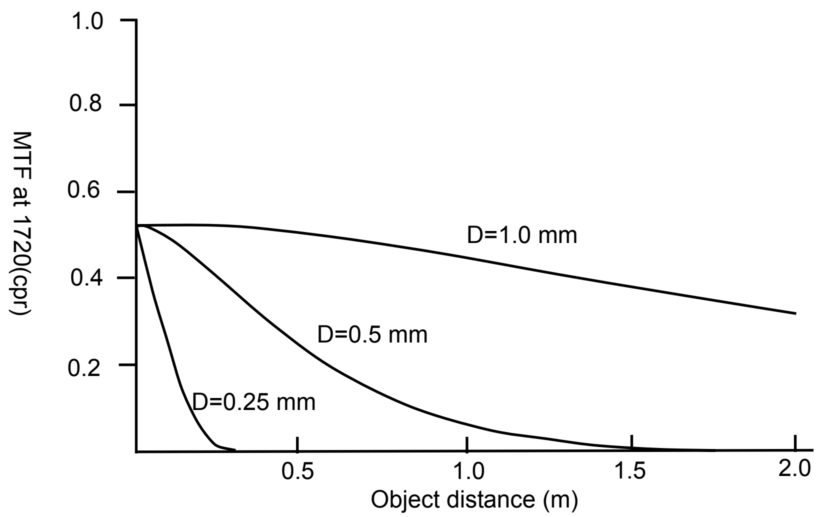


Fig. 9: MTF responses at visual-limit spatial frequency (1720 cpr)

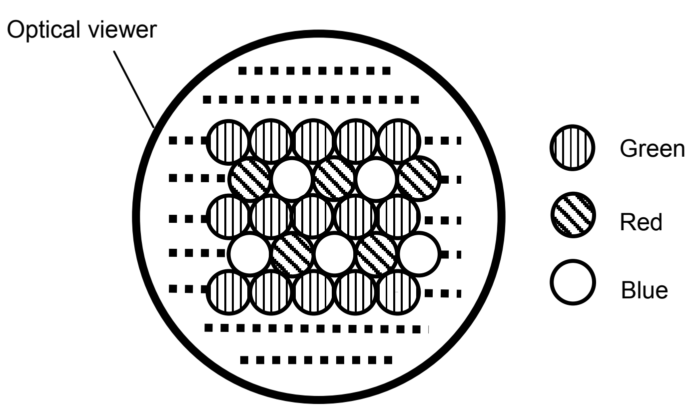


Fig.10: An example of the color filter arrays on the incident plane and the exit plane with the same structure.

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