

Teaching wavepackets propagation via ultrashort pulses of light.

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ABSTRACT.

Here we report and discuss the strategy of using the study of present ultra-short laser pulses to develop a better understanding of wavepackets and their propagation. It is well-known that in spite of its relevance and ubiquity this subject does not receive in modern optics courses the extra attention it deserves, very much in the same way that it happens in introductory physics courses or even in quantum mechanics courses. Notwithstanding, the subject has become very important both in applied optics and quantum optics, in multiple ways. For instance the generation and applications of femtosecond laser pulses, the exploitation of laser pulses as a tool in quantum control, the propagation of solitons in optical fibres, or even the propagation of light pulses in rather special media such as a Bose-Einstein condensate. A set of cases taken from applications in ultrashort laser pulse optics, non-linear optics, and optics communications, can be used to present wavepackets physics and the associated transform calculus related to the models involved. The set of cases can be also illustrated with simulations in which optical phase is seen to play the crucial role. A comparison with the traditional way of teaching wavepackets is presented.

Keywords. Ultrashort pulses, teaching of modern optics, laser pulses, femtosecond pulses, physics education, nonlinear propagation of pulses.

1. INTRODUCTION.

Wave models and wave concepts are frequent in the physics curriculum, not just in optics. The concept of plane wave is the “model”, or prototype, for teaching light, sound and at a higher level introducing quantum mechanics. But in every single course the time always come when key questions about what real waves look like have to be addressed. Then the teaching of waves have to be done with great care. For instance, the impossibility of generating the simplest one dimension-plane wave forces the introduction of the important concepts of a *wavepacket* (a term coined by Schrödinger) and its width. Although initially defined as mathematical objects, wavepackets are not enteletz. Groups of waves, that is wavepackets, are as real as the moon is, and can be even observed with bare eyes in Nature. Notwithstanding wavepackets are not receiving sufficient attention, neither in the teaching of optics nor in the rest of physics teaching. The misconceptions, and student failures that this lack of attention are creating cannot continue to be ignored. Consider for instance the various concepts that can be used to define the speed (or velocity) of a wave¹: the *phase speed*, the *group speed*, the *front speed* and the *signal speed*. Today, one may also consider the possibility of experimenting with an electro-magnetic wavepacket whose group speed can not only be much more less (say 1 cm/s !) than the famous constant speed of light c , but whose group speed can even become negative!². For instance to be able to follow the recent and notorious experiments of *superluminal light propagation*² and *slow light*³, both the students and scholars need to be familiarized with those four concepts. Of course the related concept of wavepacket *dispersion* in a given medium is another one of the teaching problems. A possible strategy to solve the difficulties once for all is to introduce and exploit the teaching of *ultrashort* pulses of electro-magnetic waves at an earlier stage in the curriculum.

Ultrashort pulses^{4,5} of laser light are quasi-coherent light signals lasting from about tens of picoseconds ($1 \text{ ps}=10^{-12} \text{ s}$) down to a few femtoseconds ($1 \text{ fs}=10^{-15} \text{ s}$). Although very difficult to produce in the laboratory, light pulses in the femtosecond regime are becoming a key tool in a number modern technologies.

Such pulses only contains a few wavelengths of the oscillating electric (or magnetic) field. In its simplest representation, in the temporal domain, one writes the electric pulse *waveform*, or wavepacket, as a complex amplitude function $E(t)$ times a *carrier signal* of angular frequency ω_0 , that is, $E(t)\exp(i\omega_0 t)$. Apart from being usually generated using lasers, ultrashort pulses are not ordinary light signals; their generation, or creation, and their measurement and applications, constitute a sophisticated technology of the last three decades. A good portion of the physics background of such pulses lies well within the curriculum of honours physics and electrical engineering degrees. For instance, in the study of ultrashort pulses ample use is made of Fourier Analysis. The width $\Delta\omega$ of the Fourier spectrum $E(\omega)$ of a pulse is related to the width Δt of the wavepacket by a well-known theorem of Fourier Analysis. One quickly becomes aware that the subject provides fertile ground and an excellent *scenario* within the physics curriculum, for the teaching/learning process of concepts such as dispersion, group velocity dispersion (GVD), and non-linear optical effects such as the optical Kerr effect. The generation of short light pulses by *pulse compression* is frequently achieved with just a short length of an optical fibre, and a pair of diffraction gratings. Ultrashort laser pulses is indeed a kind of unifying physics subject with which a large number of relevant concepts and principles can be introduced and illustrated with a significant economy of effort, and which certainly ensures a strong motivation in students. The measurement and diagnostics of ultrashort pulses⁶ is by itself a fascinating subject. Many motivating and fundamental questions can be posed when presenting ultrashort light pulses to students. How many photons reside in a 1 fs long ultrashort pulse of a given energy?. What clock can be used to measure a pulse lasting only a few fs?. What is the length of the shortest wavetrain we can produce?. What is the theoretical width of the spectrum $E(\omega)$ of such pulse?. How a femtoseconds-long pulse is related to the time-energy uncertainty relation $\Delta E\Delta T \geq \hbar/2$?. Being so short: Can ultrashort pulses produce interference effects?. Note that pulse propagation has attracted previous attention of physics lecturers since decades ago⁸. The subject seems now to be ripe for its full exploitation in the physics curriculum of the XXIst. century.

2. GENERATION OF LIGHT PULSES SHAPES

The concept of electro-magnetic wavepacket, or pulse, arises naturally within wave theory when superposition of harmonic waves is considered in the time domain (interference in time). For any pulse waveform the so-called *group speed* can be defined. It is the speed with which the pulse propagates. However, in general within a given *dispersive optical medium* the wave components of the pulse propagate at different speeds because the propagation constant β of the wave in the medium is a function of the component frequencies (ω) and can be expanded as a Taylor series:

$$\beta(\omega) = \beta_0 + \beta_1 (\omega - \omega_0) + \beta_2/2 (\omega - \omega_0)^2 + \dots, \quad (1)$$

where the different constants are given by the derivatives of the propagation constant evaluated at $\omega = \omega_0$,

$$\beta_{(n)} = [d^n \beta / d\omega^n], \quad (2)$$

The dominant terms of the propagation constant expansion (1) are the first and the second ones. The speed (v_g) of the pulse envelope is given by $1/\beta_1$. The *group velocity dispersion* (GVD) is instead determined by the second coefficient β_2 . Both constants are related to the refractive index of the medium. The second coefficient β_2 is related to the second order derivative of the medium refractive index n with respect to wavelength $d^2 n / d\lambda^2$. When the pulse propagates in a *dispersive medium* it undergoes GVD, and therefore the (temporal) width of the pulse changes. The ideal pulse waveform is the *gaussian*:

$$E(t) = \exp(-at^2), \quad (3)$$

with an intensity profile whose half-width τ_G is given as a function of the pulse parameter a by,

$$\tau_G = [2 \ln(2)/a]^{1/2} \quad (4)$$

It is said to be *diffraction limited* by analogy with the case of a perfect optical aperture. The analogy arises from the similarities of the time-based theory of short pulses with the diffraction phenomena which beams of light undergo when they propagate in space (this is known as the *spatio-temporal analogy*). Either as a consequence of the physics process by which the pulse is created (e.g. by the features imposed on the pulse by the pulsed laser itself), or as a consequence of its propagation through a given optical component, a pulse acquires unexpected shapes and unexpected phase dependences. Its intensity profile usually broadens as the distance travelled by the pulse increases (even if it maintains its initial gaussian profile). One of the possible resulting pulse shapes is the *chirped gaussian*, whose complex amplitude is written in terms of two parameters a and b by,

$$E(t) = \exp(-a + i b) t^2. \quad (5)$$

In this case the time-dependent phase function φ of the pulse is given by,

$$\varphi(t) = \omega_0 t + b t^2. \quad (6)$$

It is a very interesting case. The time dependent instantaneous frequency relation,

$$\omega = d\varphi/dt = \omega_0 + 2b t, \quad (7)$$

shows that the carrier frequency shifts *linearly* with time under the pulse waveform (Figure 1). Of course, the frequency shift is controlled by the *linear chirp parameter* b (see equation 6) of the phase function. Chirping is a trick well known to bats and birds, which emit chirped ultrasound pulses to locate prey in the air or define their species characteristic song with chirps, respectively. For *positive* chirp, the chirp parameter b is greater than zero. This implies that the frequency decreases towards the trailing edge of the pulse (Figure 1). Chirping is seen therefore to be dictated by the phase information in the pulse. This is also true of most pulse shapes, in spite of their differences. For instance, *3rd order* or *cubic chirp* (Figure 2) arises sometimes in a pulse. This is defined by the new phase function φ ,

$$\varphi(t) = \omega_0 t + c t^3. \quad (8)$$

It can be seen^{5, 6} that cubic chirp can be imposed on a pulse after narrow-band amplification of the pulse. In such case the leading and trailing edges of the pulse have higher frequencies than at the centre of the pulse. This sort of pulse is depicted in Figure 2, where the central portion of the pulse shows the slower frequencies of the pulse. Another interesting case occurs when the pulse intensity $I(t) = |E(t)|^2$ is sufficiently high to force the medium of propagation to behave non-linearly. For instance it is frequent that in such cases the medium refractive index changes from the usual (tabulated) value n_0 to a new value $n = n_0 + n_2 I$, the latter is linearly dependent on the travelling pulse intensity. This imposes on the travelling pulse the important non-linear effect called *self-phase modulation* (SPM). In such case the phase function of the pulse becomes,

$$\varphi(t) = \omega_0 t + s |E(t)|^2. \quad (9)$$

It may be seen that for positive values of the parameter s , lower frequencies are generated in the leading edge of the pulse, while higher frequencies in the trailing edge.

Laboratory pulsed lasers (e.g. NdYag lasers) generate pulses of quasi-coherent light, which are roughly tens of nanoseconds long, and contain a few millijoules of energy. Pulsed semiconductor lasers normally emit nanosecond long pulses that already display either form of linear chirp (positive or negative). To convert such pulses to the ultrashort regime, the pulses need to be further *compressed in time* using different techniques. The pulse may be allowed to propagate in a short length of a linearly dispersive medium. The medium induces chirp on the pulse due to group velocity dispersion within the medium. If this second chirp happens to be the opposite of the initial chirp of the pulse then the pulse becomes narrower. This is easily explained: the frequency components of the pulse travel at different speeds under the electric field envelope of the pulse. In a positively chirped pulse the frequency decreases towards the trailing edge. GVD can then be exploited to slow down the blue-shifted components at the leading edge of the pulse, which then is forced to coincide in time with the trailing edge of the pulse. The exact cancellation occurs at an exact specific distance. The trick can be neatly performed propagating the pulse along the adequate length of an optical fibre. Pulse compression can also be achieved using an optical fibre in which non-linear SPM cancels the effect of GVD giving a pulse whose shape can not only be compressed but also maintained. Finally we mention the interesting case of the *Fibre-Grating Pair Compressor* in which the pulse travels first along a short length of optical fibre (Figure 3). Within the fibre the pulse acquires a small amount of positive chirp via the interplay of GVD and SPM. Afterwards the pulse is externally compressed using a pair of parallel diffraction gratings, which provide anomalous GVD to the pulse. As in any grating different frequency components of the pulse are diffracted in slightly different directions. They thus undergo different delays when travelling in between the gratings. For a negatively chirped pulse, the red shifted components in the leading edge are delayed more, catching up with the blue-shifted components in the trailing edge of the pulse. The result is once again a compressed pulse. The scheme is mostly used to compress pulses in the visible and near-infrared region of the spectrum. Typically pulses which are tens of femtoseconds long are obtained.

It is relatively easy to simulate the propagation of a given pulse in a dispersive medium, at least if the medium behaves linearly. In effect one can easily obtain the Fourier spectrum of the input pulse (i.e. its Fourier Transform). The spectrum is then multiplied by the frequency-dependent propagation function of the linearly behaved medium, that is $\exp[-i\beta(\omega)z]$ where z is the distance travelled by the pulse in the medium. Finally one has only to take the inverse Fourier Transform of the propagated spectrum to obtain the output pulse⁶. Available commercial software (e.g. *Matlab*) contains procedures for Fourier transforming the signals. In this way it is relatively straightforward to simulate propagation of many kind of pulses in the linear medium. At a more advanced level the nonlinear propagation of narrow pulses in a moderately nonlinear medium can also be simulated. The basic propagation or pulse motion equation that has to be solved is the so-called *nonlinear Schrödinger equation*⁶. With such formalism even the case of propagation of a short pulse in a length of optical fibre (as in Figure 3) can be simulated.

4. MEASURING ULTRASHORT PULSES.

It is an useful exercise to think about how to measure the “length” of an ultrashort pulse. The simplest way to measure the temporal width of a pulse is to correlate the pulse with itself. A beam splitter can be used to separate the initial pulsed beam in two equal beams, of half the initial intensity, which are then brought together in a crystal block (say potassium dihydrogen phosphate or KDP) which gives a second harmonic generation signal. A variable delay line is set up in the path of one of the two beams to produce the correlation in time. It is relatively simple, but the phase of the pulse is lost. Nice clever techniques, such as the Frequency-Resolved Optical Gating^{7,10}, can give both the intensity profile and the phase of the pulse through the measurement of a kind of spectrogram of the pulse (*time delay-wavelength plot*) and the application of a computer algorithm. The shortest pulse thus far generated measured only a few femtoseconds long.

5. APPLICATIONS OF ULTRASHORT PULSES

Ultrashort pulses have found numerous important applications⁹⁻¹². They are indeed very important tools for high-resolution atomic and molecular spectroscopy, and also for understanding about fast non-linear processes in key materials of modern technologies¹¹. They have been applied in the study of fast processes in photosynthesis and vision phenomena. Ultrashort pulses of laser light have opened for the first time the possibility of performing so-called *coherent quantum control* of chemical dynamics. It is known that the timescale for making and breaking molecular bonds in ordinary chemical reactions is set by the vibrational period of the molecules. Such period is precisely of the order of tens of femtoseconds. Only femtosecond laser pulses can be used “to resolve” the temporal details of molecular bond formation and rearrangement^{11, 12}. Chemical equilibrium in molecular reactions is in principle open to manipulation by using well known *pump-and-probe* techniques, this time with femtosecond laser pulses. The *pump* pulse initiates the reaction (by exciting one of the reactants to the proper quantum state, lying at a higher energy level). The phase and intensity of the laser pulse are the key factors in the preparation of the particular excited state so as to enhance the production of a desired chemical compound. A series of weaker probe laser pulses (derived from the same laser pulse used as the pump, with a beam-splitter and a variable delay line) are then used to “observe” the ensuing fast reaction process. Sometimes the probe pulse generates quantum interference effects in the population of a given reaction product.

6. A POSSIBLE COURSE ON ULTRASHORT PULSES

Contents of a course on ultrashort light pulses can be naturally divided into: (i) Laser Pulse Representation and Parameters, (ii) The Generation of Ultrashort pulses, (iii) Measurement and Diagnostics, (iv) Pulse Propagation in a Linear Medium, and finally (v) Applications of Ultrashort Laser Pulses. The best scenario would be a proper course (some 50 hours) on Ultrashort Pulses. Alternatively the subject can be presented in about 12 hours, as a piece of Modern Optics or a Non-linear Optics Course. Standard courses on Vibrations and Waves and Fourier Transform analysis, or at least formal studies of these subjects in parallel are requisites. We have tested the exploitation of ultrashort pulses as a teaching strategy a couple of times with some success. The low number of students in the courses, and a long time period of separation between the two occasions have impeded a rigorous or careful study of the possible differences with respect to standard courses on waves and vibrations. Nonetheless, the opinions and the motivation of the students, the marks obtained by them, and the level of difficulty tackled by the students in the exams show that the idea of teaching ultrashort pulses is definitely sound. At least not a single complaint was registered. When detailed knowledge of Fourier Analysis relations, or skill operating with such transform, was demanded we did observe some student limitations, these limitations of course have nothing to do with the physics interpretations involved in the propagation of the pulses. As a whole we can safely report that teaching on these ultrashort pulses was not only successful but actually fun for the total number of students who took the two courses.

7. DISCUSSION.

We have presented firm evidence that ultrashort pulses of laser light should now become part of the curriculum of honours physics and electrical engineering degrees; either as a proper course, or as a part of a larger course in non-linear or modern optics. In spite of its recent development, it is a subject mostly based on standard knowledge presented in linear and non-linear optics. In fact there is an almost perfect analogy between diffraction phenomena in space domain and pulse propagation in the time domain (spatio-temporal analogy). The mathematics required is not that much advanced, basically Fourier Analysis. The subject has a rich conceptual content. A number of interesting simulations of pulse propagation can be easily done, particularly if the propagation is in a linear medium. The most interesting nonlinear propagation case can also be tackled with available commercial software (a nonlinear Schrödinger equation has to be numerically solved). The applications are diverse, and include the whole fast developing field of ultrafast phenomena, including the important case of coherent quantum control.

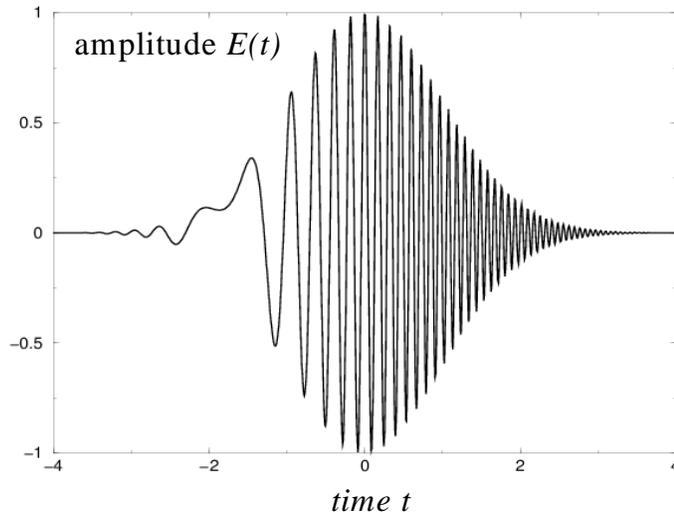


Fig. 1. Positive-chirped gaussian pulse of complex amplitude $E(t)$ and parameters $\omega_0=36$, $a=0.5$, $b=10$.

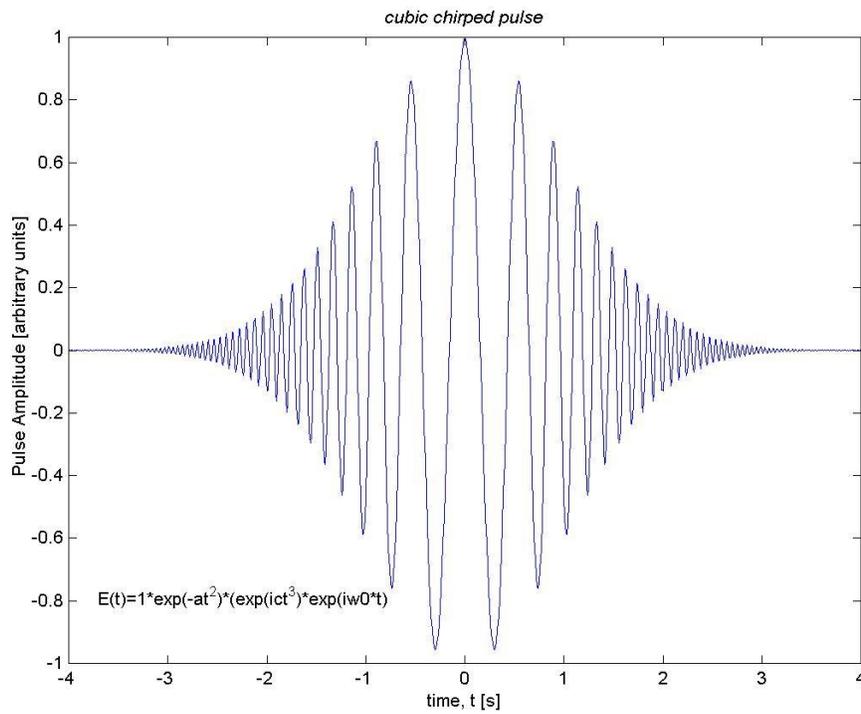


Fig. 2. A gaussian cubic-chirped pulse ($a = 0.5$, $c = 5$, $\omega_0 = 36$)

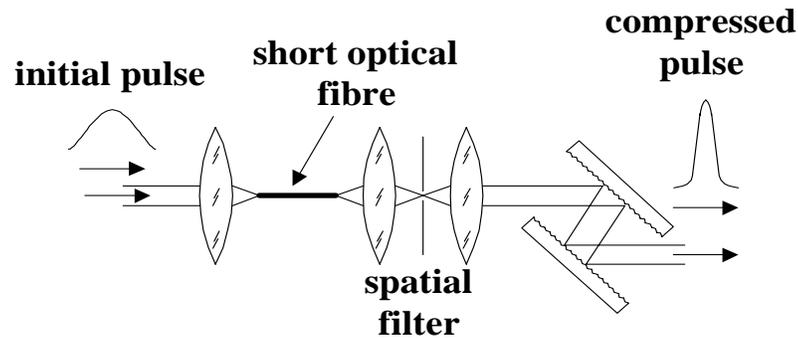


Figure 3. Fibre optics-diffraction grating set-up for pulse compression.

7. REFERENCES

1. L. Brillouin, *Wave Propagation and Group Velocity*, Academic, New York (1960).
2. K. J. Wang, A. Kuzmich and A. Dogariu, "Gain assisted superluminal propagation", *Nature* **406**, 277-279 (2000).
3. L. V. Hau, S. E. Harris, Z. Dutton and C. H. Behroozi, "Light speed reduction to 17 meters per second in an ultracold atomic gas", *Nature* 594-598 (1999).
4. S. L. Shapiro (Ed.), *Ultrashort Light Pulses*, Springer-Verlag, Berlin (1977).
5. S. A. Akhmanov, V. A. Vysloukh and A. S. Chirkin, *Optics of Femtosecond Laser Pulses*, A.I.P. Press, New York, 1992.
6. A. E. Siegman, *Lasers*, University Science Books, Mill Valley, California (1986).
7. R. Trebino and D. J. Kane, "Using phase retrieval to measure the intensity and phase of ultrashort pulses: frequency-resolved optical gating", *J. Opt. Soc. Am. A-10*, pp. 1101-1101, 1993).
8. J. Jones, "On the propagation of a pulse through a dispersive medium", *Am. J. Phys.*, **42**, pp. 43-47, 1974.
9. G. R. Fleming and A. E. Siegman, *Ultrafast Phenomena V*, Springer-Verlag, Berlin, 1986.
10. K. W. DeLong, C. L. Ladera, R. Trebino, B. Kohler and K. R. Wilson, "Ultrashort Pulse measurement using non-instantaneous nonlinearities: Raman effects in frequency resolved optical gating", *Opt. Lett.*, **20**, pp. 486-488, 1995.
11. Search and Discovery, *Phys. Today*, **52**, No. 12, pp.19-21, 1999.
12. P. Brumer and M. Shapiro, "Quantum interference in the control of molecular processes", *Highlights in Quantum Optics*, B. Stoicheff and D. Walls (Eds), *Phil. Trans. R. Soc. London A*, pp. 2409-2412, (1997)