Invited Paper

A Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons

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ABSTRACT

John Bell's theorem of 1964 states that local elements of physical reality, existing independent of measurement, are inconsistent with the predictions of quantum mechanics (Bell, J. S. (1964), Physics (College. Park. Md). **1** (3), 195). Specifically, correlations between measurement results from distant entangled systems would be smaller than predicted by quantum physics. This is expressed in Bell's inequalities. Employing modifications of Bell's inequalities, many experiments have been performed that convincingly support the quantum predictions. Yet, all experiments rely on assumptions, which provide loopholes for a local realist explanation of the measurement. Here we report an experiment with polarization-entangled photons that simultaneously closes the most significant of these loopholes. We use a highly efficient source of entangled photons, distributed these over a distance of 58.5 meters, and implemented rapid random setting generation and high-efficiency detection to observe a violation of a Bell inequality with high statistical significance. The merely statistical probability of our results to occur under local realism is less than $3.74 \cdot 10^{-31}$, corresponding to an 11.5 standard deviation effect.

Keywords: Bell test, entangled photon source, polarization entanglement, signifcant-loophole-free test, local realism, Bell inequality, polarization measurements

1. INTRODUCTION

In 1935, Einstein, Podolsky and Rosen (EPR) reasoned that the quantum mechanical wave function can not completely describe physical reality.[1] This argument can be easily understood using Bohm's Gedankenexperiment:[2,3] Consider a spin-0 molecule, that gets split up into two spin-$\frac{1}{2}$ atoms and distributed to two space-like

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separated measurement stations, “Alice” and “Bob”. Whenever Alice measures the spin of her atom, she knows that – due to angular momentum conservation – the spin that Bob measures has to add up to 0 with her measurement outcome, whether she measures spin $+\frac{1}{2}$ or $-\frac{1}{2}$. In other words, her measurement outcome makes it possible for her to predict with certainty Bob's outcome. Alice' and Bob's result will always exhibit this strong correlation whenever they have chosen to measure in the same measurement basis. Since there is no interaction between the two systems anymore, it might be reasonable to believe that the measurement outcomes have been predetermined, but the quantum mechanical description does not carry any information about the outcomes. This is the essence of the argument by EPR that the quantum state as a description of physical reality can not be complete.[1]

Bell's theorem of 1964 shows that *local realism*, the worldview under which physical influences are limited by the speed of light and measurement outcomes are defined prior to and independent of measurement, is inconsistent with the predictions of quantum mechanics.[4] Specifically, correlations between measurement results from distant entangled systems would be smaller under the assumption of *local realism* than predicted by quantum mechanics. This is expressed in Bell's inequalities. Since quantum mechanics predicts a violation of the inequality for the results of certain measurements on entangled particles, Bell's inequality can be used here to rule out philosophical standpoints based on experimental results. Indeed, violations have been measured employing versions of Bell's inequalities.[5–7]

Do these experimental violations invalidate local realism? That is not the only possibility. The experiments violating Bell's inequality required extra assumptions, and therefore opened loopholes that in principle still permit that the measured data can be explained using a local realist model.

2. LOOPHOLES

The *locality loophole* (or *communication loophole*) is left open if the setting choice or the measurement result of one side could be communicated at the speed of light in vacuum or slower to the other side in time to influence the measurement result there. In order to close this loophole, it is necessary to space-like separate each local measurement from the distant setting choice as well as from the distant measurement. This can be guaranteed by independently choosing the measurement settings on both sides so quickly that no physical signal can pass information about the chosen setting or the measurement result to the other side in time to influence the measurement.

The *freedom-of-choice loophole* regards the possibility of influences on the setting choices from any combination of *hidden variables* and/or other factors within the backward light-cone of the setting choice. Here, *hidden variables* represent “any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR.”.[8] In order to address this loophole, it is necessary to make specific assumptions about the origin of these hidden variables and generate the setting choices independently from past events and space-like separated from the hidden variables. We make the assumption that the hidden variables are created not before the emission event of the entangled photon pair.

The *fair-sampling loophole* is about the idea, that a small sub-ensemble of all entangled particles could in principle be non-representative for the entire ensemble of entangled particles.[9] For example, it is imaginable that the detected sub-ensemble could violate Bell's inequality while the entire ensemble does not. It is possible to close this loophole by detecting the entangled particles with a sufficiently high efficiency.

The *coincidence-time loophole*[10,11] exploits the assumption that the timing statistics is the same for all detector clicks. This particularly applies to experiments in which the identification of pairs is done via a moving coincidence window. One way to avoid this loophole is to make a pulsed experiment with locally defined time slots.

The *memory loophole*[12] corresponds to the assumption that experimental trials are identical and independent (iid). In principle the outcomes of a specific trial could depend on all previous settings and outcomes since these are not space-like separated anymore. Exploiting this loophole, the statistical significance of a violation can be altered. This loophole can be closed by avoiding the iid assumption in the data analysis.

Many experimental Bell tests have been performed[6,7,13–25] closing individual loopholes. For example, Aspect *et al.*'s 1982 experiment[7] first employed rapid switching in the measurement settings; Weihs *et al.*[13] improved this with fast random switching; Scheidl *et al.*[18] addressed the freedom-of-choice and locality loopholes in 2010 while Handsteiner *et al.*[25] improved on that; Rowe *et al.*[14] were first to close the fair-sampling loophole in 2001 and were followed by several experiments in a variety of systems.[15,17,20–22] It has recently become possible to address all aforementioned loopholes in a single experiment.[22–24,26] In this paper, we report the violation of a Bell inequality while closing all aforementioned loopholes in a single experiment with high statistical significance. Our experiment therefore strongly supports the claim that nature cannot be described within the framework of local realism.

3. SETUP

In Fig. 1(a), the experimental setup is illustrated. The source of polarization-entangled photon pairs made use of spontaneous parametric down-conversion (SPDC) in a periodically poled nonlinear crystal (ppKTP). The polarization entanglement was facilitated using a Sagnac configuration[27,28] and optimized focusing parameters for high heralding efficiency.[29,30] With single-mode fibers, the photons were distributed to the two measurement stations, "Alice" and "Bob" [Fig. 1(c)] to perform polarization measurements on them. While the photons were on their way towards the measurement station, a random number generator[31,32] (RNG) made a choice between two linear polarization angles which were implemented by an electro-optical modulator (EOM) that acted as a polarization rotator in front of a polarizing beam splitter. The horizontal output of that beam splitter was connected to a transition-edge sensor (TES) single photon detector.[33] The signal from the TES was amplified by several cryogenic[34] and room-temperature amplifiers, digitized and recorded locally on a hard drive together with the time stamp and result of the basis choice.

3.1 Closing the Freedom-of-choice and Locality Loopholes

In order to close the freedom-of-choice and locality loopholes, a very specific space-time arrangement was necessary as discussed above in sec. 2. In the space-time diagram of the experiment [Fig. 2], three events are of particular importance:

1. The *emission*, which is depicted as a blue dot at the origin of the space-time diagram. This is the earliest possible point in time that a photon pair could have been emitted, since it corresponds to the leading edge of the pump pulse reaching the nonlinear crystal.

2. The *setting choice* was performed by the random number generator[32] during the time interval depicted by the green bars. During this interval, four random bits were generated and their parity was determined as setting choice. Each random bit corresponds to an evaluation of the – due to spontaneous emission randomized – phase between consecutive laser pulses. The more bits are generated in the allowed time interval, the smaller is the finite predictability.[31,32]

3. The *measurement* has to be performed within the time interval depicted as red bars to be space-like separated from the measurement outcome on the other side. The beginning of the interval is the time at which the photon passes the polarizing beam splitter after the EOM. Within this time interval, the photon is absorbed by the TES single photon detector and the electrical signal behind the SQUID has risen out of the noise-level to be discriminated and used for time stamping.

All relevant delays were characterized using an oscilloscope and a fast photodiode relative to a 1 MHz clock which was also used to control the pump laser and EOM. This clock was phase stable to a 10 MHz master oscillator which kept the time tagging devices, digitizer cards and random number generators synchronized.

3.2 Closing the fair-sampling Loophole

The closure of the fair sampling loophole can be observed in the measured data. It is the cleanest way to use an inequality that can be derived without the fair-sampling assumption. This applies to both the Clauser-Horne[5] and Eberhard[35] inequality which can be violated at a system heralding efficiency of larger than 2/3. We used a

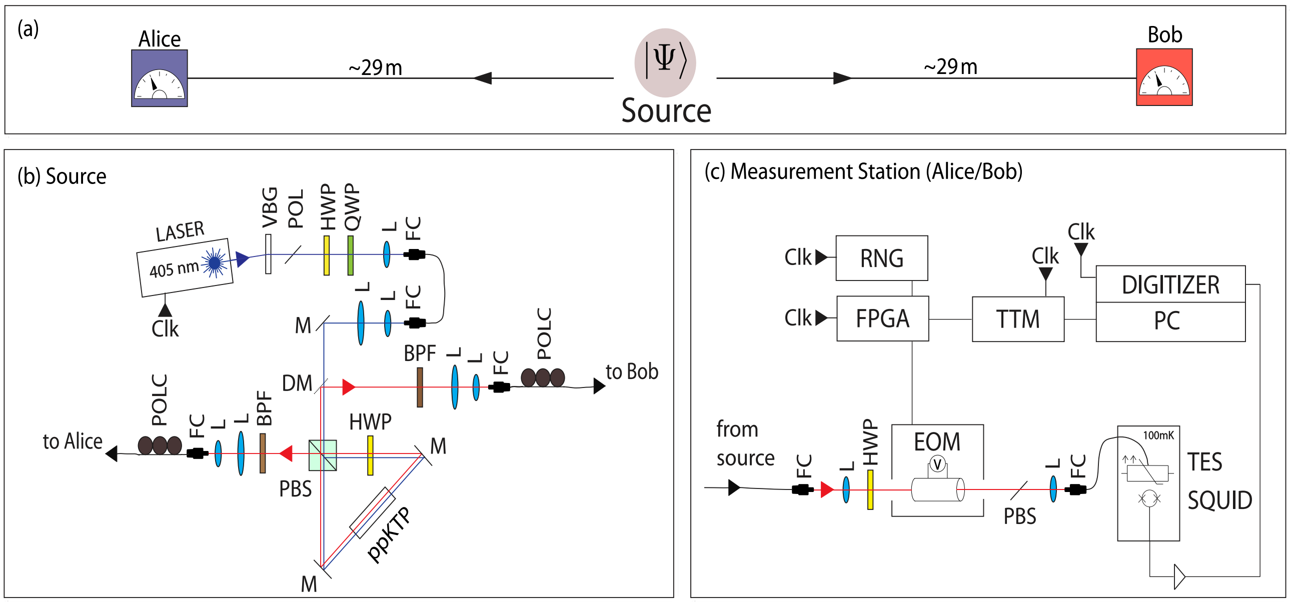

Figure 1. (a) Overview of the setup. (b) Source: A pair of polarization-entangled photons was shared between the two identically built and spatially separated measurement stations *Alice* and *Bob* (distance $\approx 58\,\mathrm{m}$), where their polarization was analyzed. The source employed type-II spontaneous parametric down-conversion in a periodically poled crystal (ppKTP). The pump laser emitted 12 ns long pulses (FWHM) which were filtered spectrally by a volume Bragg grating (VBG, FWHM: 0.3 nm) and spatially by a single-mode fiber. The ppKTP crystal was pumped from both sides in a Sagnac configuration to generate polarization entanglement. At the polarizing beam splitter (PBS), each pair was divided and afterwards collected into two different single-mode fibers leading to the measurement stations. (c) Measurement stations: One of two linear polarization directions was selected for measurement, as controlled by an electro-optical modulator (EOM), which acted as a switchable polarization rotator in front of a plate PBS. Customized electronics (FPGA) sampled the output of a random number generator (RNG) to trigger the switching of the EOM. After passing the transmitted output of the plate PBS, the photons were coupled into a fiber to the TES. The signal of the TES was amplified by various amplifiers, digitized, and recorded together with the setting choices on a local hard drive. The laser and all electronics related to switching/recording were synchronized with clock inputs (Clk). Abbreviations: BPF: band-pass filter; DM: dichroic mirror; FC: fiber connector; HWP: half-wave plate; L: lens; M: mirror; POL: polarizer; POLC: manual polarization controller; QWP: quarter-wave plate; SQUID: superconducting quantum interference device; TES: transition-edge sensor; TTM: time-tagging module. Figure adapted from.[23]

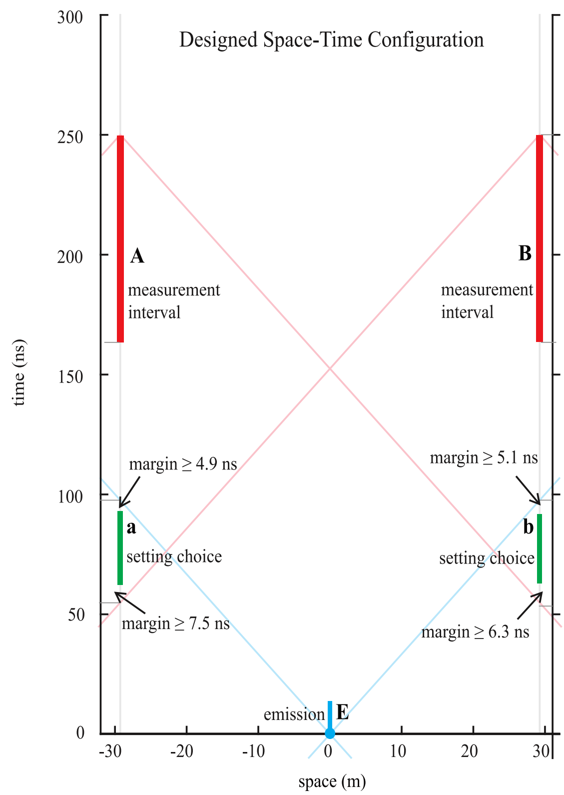

Figure 2. Space-time diagram depicting experimental design and construction to scale. The diagonal lines indicate the light cones, i.e. the speed of light in vacuum. The interval of pair emission is represented in light blue (E), corresponding to the duration of the pump laser pulse. The choice and application of measurement settings is confined to the green bars a and b respectively, and the measurement takes place within the duration of the red bars A and B. The setting choice interval is limited from the one side by the forward light cone of the earliest possible emission event and on the other side by the backward light cone of the endpoint of the distant measurement interval. Figure adapted from.[23]

CH-Eberhard[36] inequality which makes use of only one detector per side and considers the outcomes "+" for a detection event and "0" for no detection.[36,37]

$$J \equiv p_{++}(a_1, b_1) - p_{+0}(a_1, b_2) - p_{0+}(a_2, b_1) - p_{++}(a_2, b_2) \leq 0 \tag{1}$$

For each trial, Alice choses between a_1 and a_2 and Bob choses between b_1 and b_2. For example, $p_{+0}(a_1, b_2)$ is the probability that Alice detected a photon and chose the angle a_1 and Bob has no detection event and chose the angle b_2. Both of them write down their outcomes "+" or "0" for each trial and compare their data after the experiment to estimate the probabilities and evaluate the inequality. This inequality is maximally violated by non-maximally entangled states of the form:

$$|\Psi\rangle = \frac{1}{\sqrt{1+r^2}}(|V\rangle_A|H\rangle_B + r|H\rangle_A|V\rangle_B) \tag{2}$$

The optimal parameter r was found using numerical simulations based on a quantum mechanical model[38] and depends on the system efficiency, the visibility and the background rate. We used a parameter of $r \approx -2.9$ and measured at the angles $a_1 = 94.4°$, $a_2 = 62.4°$, $b_1 = -6.5°$, $b_2 = 25.5°$ for approximately 3510 seconds.

3.3 Closing the Coincidence-time and Memory Loopholes

The coincidence-time loophole was avoided by using a pulsed experiment with locally defined time slots. Therefore, the identification of coincident photons does not rely on any method that opens the coincidence-time loophole.

The assumption that the experimental trials are independent and identical was avoided in order to close the memory loophole.[12] The statistical significance was computed assuming full experimental memory.[23,36]

4. RESULTS

We characterized the system using a maximally entangled state ($r = -1$ in Eq. 2) and found a visibility of $> 99\%$ in both the diagonal and the HV-basis. The total system efficiency (i.e. the ratio of twofold coincidence events per single counts) was approximately 78.2% in the Alice arm and 76.2% in the Bob arm. Approximately 3500 pairs were created per second in the source. We determined a J value of $7.27 \cdot 10^{-6}$. A p-value of $3.74 \cdot 10^{-31}$

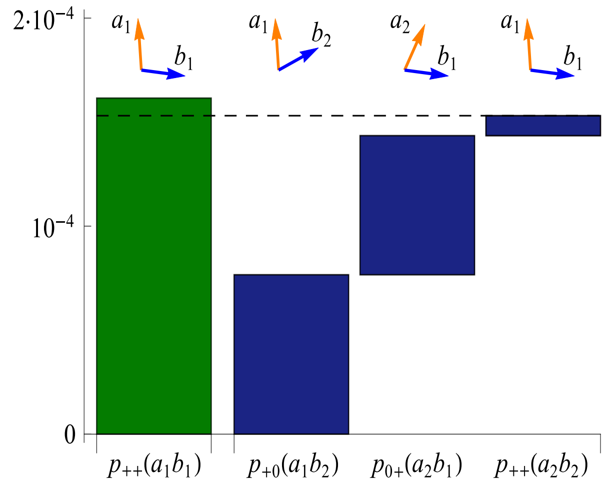

Figure 3. Bar chart of the four probabilities that enter the inequality (Eq. 1). The green (left) bar representing $p_{++}(a_1b_1)$ outweighs the sum of the other three red bars, therefore the J-value is positive and the CH-Eberhard inequality is violated. Figure adapted from.[23]

was computed under full experimental memory[12,39,40] while taking into account the finite predictability of the random number generators.[36] This is the purely statistical probability of our observed violation to be the result of statistical fluctuations under local realism. Given the very small probability, it should be mentioned that the confidence in this experiment is limited in general by other sources of errors including systematic and human mistakes rather than by the statistical significance.

5. SUMMARY

We demonstrated a strong violation of local realism with high statistical significance. We space-like separated the emission event of the down-conversion from the setting choice and the setting choice from the measurement using state-of-the-art random number generators. We closed the fair-sampling loophole with a very high system heralding efficiency. We also closed the coincidence-time loophole by using locally defined time slots and the memory loophole by adequate statistical analysis. Our experiment provides strong support for the viewpoint that local realism is untenable.

The freedom-of-choice loophole was closed up to a reasonable point in time: The production of the entangled photon pair. However, this is just a few hundred nanoseconds before the measurement. What if the hidden variables have been created a long time before the experiment? It is possible to use setting choices that have been produced and space-like separated for billions of years by using light from different quasars on opposite sides of the night sky.[41] Handsteiner *et al.*[25] used basis settings derived from the light of Milky Way stars to push back the time when the hidden variables could have been created by ~ 600 years. Further steps could be to use photons from quasars that are space-like separated since the period of cosmic inflation and also close the fair-sampling loophole at the same time.

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