# Linear and nonlinear effects in optical transmission

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## ABSTRACT

The transmission of high-bandwidth data over optical fiber is impaired by a variety of linear and nonlinear effects due to the inherent properties of the transmission medium. In this tutorial, we will explain how these physical effects affect the data transmission. We will also discuss the interdependence between these effects and give some rules of thumb to estimate their relative impact.

**Keywords:** Optical transmission, chromatic dispersion, Kerr effect, self-phase modulation, cross-phase modulation, four-wave mixing, stimulated Raman scattering, stimulated Brillouin scattering

## 1. INTRODUCTION

The transmission of high data rate signals over long distances is limited by a number of fiber properties that act together to distort the optical signal. Chromatic dispersion (CD) distorts the relative phase information of the signal and leads to temporal distortion of the bit pattern. The non-linear Kerr effect induces self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM) between different channels of a WDM system. Stimulated Brillouin Scattering (SBS) noisily scatters part of the signal power, Stimulated Raman Scattering (SRS) transfers power from channels at one end of the signal spectrum to other channels. In this paper, we will further describe these effects and give rules of thumb to estimate their relative impact on optical data transmission.

In the absence of chromatic dispersion and non-linear effects, the signal would propagate with constant pulse shape, only impacted by the fiber attenuation. This propagation can be described in the time domain by a simple version of the Schroedinger equation[1]:

$$\frac{\partial A(z,t)}{\partial z} = -\frac{\alpha}{2} A(z,t) - \beta_1 \frac{\partial A(z,t)}{\partial t}, \tag{1}$$

where A(z,T) is the slowly varying signal pulse amplitude with $|A(z,T)|^2 = P(z,T)$ is the instantaneous power of the signal, $\alpha$ is the fiber attenuation coefficient, and $\beta_1$ is the group propagation constant $1/v_g$. Usually, the second term on the RHS of Eq. (1) is eliminated by introducing a frame of reference, moving with the pulse at group velocity $v_g$, and including the time coordinate T:

$$\frac{\partial A(z,T)}{\partial z} = -\frac{\alpha}{2} A(z,T) \tag{2}$$

The solution for this equation is a signal with constant shape and exponentially decreasing amplitude. To consider non-linear effects, it often makes sense to evaluate the integral of the signal power over a fiber span. As the signal power, derived from Eq. (2), is

$$P(z) = P(0) \cdot \exp(-\alpha z), \tag{3}$$

the integral over the fiber span yields

$$\int_0^L P(z)dz = P(0) \cdot \frac{1-\exp(-\alpha L)}{\alpha} = P(0) \cdot L_{eff} \tag{4}$$

Eq. (4) defines the "effective length" $L_{eff}$ of a fiber span.

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## 2. CHROMATIC DISPERSION

While a single-frequency DFB laser, as typically used in transmission systems, can have a narrow linewidth on the order of 1 MHz, modulation of the laser light with data broadens the spectrum to a value on the order of one to two times the data rate. Modulation with return-to-zero (RZ) signals can lead to even broader signal spectra. Due to the chromatic dispersion of the transmission fiber, the different spectral signal components propagate with different velocity, and therefore the signal pulses are broadened, closing the signal eye. This pulse-shaping effect can be described in the time-domain by adding a term to the Schroedinger equation, Eq. (1). If only chromatic dispersion and fiber attenuation are considered, the propagation of the signal pulse in the z-direction is described by

$$\frac{\partial A(z,T)}{\partial z} = -\frac{\alpha}{2} A(z,T) - \frac{j}{2} \beta_2 \frac{\partial^2 A(z,T)}{\partial T^2}, \tag{5}$$

where $\beta_2$ is the fiber dispersion coefficient. The phase distortion of the signal, as introduced by the second term on the RHS of Eq. (5), increases with variation in the signal shape A.

While the fiber dispersion coefficient $\beta_2$ is used for the propagation equation description, the more practical dispersion parameter D is used in system evaluations. D is defined as the wavelength derivative of the propagation time per unit distance,

$$D = \frac{1}{L} \frac{\partial \tau}{\partial \lambda} \tag{6}$$

and is related to $\beta_2$ as

$$D = -\frac{2\pi c}{\lambda^2} \beta_2 \approx -0.8 \frac{ps}{nm \cdot km} \cdot \frac{\beta_2}{ps^2/km}, \tag{7}$$

where the approximation is valid around $\lambda = 1550$ nm. Please note the negative sign in Eq. (7). Positive dispersion values D correspond to negative values of $\beta_2$ and are called the “anomalous” dispersion regime.

Based on the wavelength domain picture of chromatic dispersion, a dispersion limit for signal propagation can be defined for an initially dispersion-free signal. If the spectral width of the modulated signal is $\Delta\lambda$ and the dispersion limit is defined by the outer edges of the signal spectrum differing in propagation time by the bit duration $T_B$, the maximum accumulated dispersion is limited by

$$D \cdot L \leq \frac{T_B}{\Delta\lambda} \tag{8}$$

When an external modulator is used, the optical bandwidth of an NRZ signal[2] is approximately 1.2 times the bit rate, B. The dispersion limit is then approximately[3]

$$D \cdot L \leq \frac{10^5}{\left( B/Gbps \right)^2} \frac{ps}{nm} \tag{9}$$

For a bit rate of 10 Gb/s, the dispersion limit is about 1000 ps/nm, corresponding to about 60 km of standard single mode fiber (SSMF) at a signal wavelength of 1550 nm. The dispersion limit is reduced by a factor of four when the bit rate is doubled.

In order to obtain longer transmission distances than this dispersion limit, several methods[2] can be used to invert the acquired signal distortion to compensate for the linear effect of fiber dispersion.

## 3. KERR EFFECT

While the refractive index of the fiber is wavelength dependent, leading to signal distortions by chromatic dispersion, it is also dependent on the instantaneous power in the fiber. This is known as the "Kerr effect".

$$n = n_0(\lambda) + n_2 \frac{P}{A_{eff}} \tag{10}$$

Here, $n_2$ is the non-linear fiber coefficient, $A_{eff}$ is the effective area of the fiber, which is the average area occupied by the field in the fiber, and P is the instantaneous signal power. A non-linearity coefficient $\gamma$ is often used to describe the non-linear properties of fiber, which includes the fiber parameter $A_{eff}$ in addition to the non-linear fiber coefficient $n_2$ and which is defined as

$$\gamma = \frac{2\pi n_2}{\lambda A_{eff}} \tag{11}$$

A typical value for $\gamma$ is about 1.3 $(\text{W-km})^{-1}$ in standard single-mode fiber. As the Kerr effect impacts the signal phase during fiber propagation, it can be included in the Schroedinger equation (Eq. (5)) as an additional imaginary term, yielding the non-linear Schroedinger equation:

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A - \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + j\gamma \left|A_{agg}\right|^2 A \tag{12}$$

While the slowly varying signal pulse amplitude A can be considered separately for each channel in a WDM system, the expression $\left|A_{agg}\right|^2$ in the third term on the RHS of Eq. (12) is the aggregate instantaneous power of all signal channels at a position z in the fiber. The part of this power related to the considered channel itself leads to self-phase modulation (SPM). Signal power components from other channels of the WDM system lead to cross-phase phase modulation (XPM), and varying power terms from beating between different signal channels lead to four-wave mixing (FWM). We will look into these effects separately in the following sections, but keep in mind that SPM, XPM and FWM all arise from the power-dependent refractive index of the transmission fiber (Kerr effect).

### 3.1 Self-phase modulation

As the refractive index depends on the instantaneous signal power, the phase that a signal acquires during propagation in the fiber depends on the time-dependent power and the shape of the signal. At the signal transitions between high and low power, the acquired phase will change, resulting in a frequency shift of the signal. The leading edge of a pulse will be shifted towards lower frequencies ("red shift"), while the trailing edge will experience a blue shift. This frequency shift alone does not constitute a signal distortion as long as all optical filters in the transmission path have sufficient bandwidth to let pass the widened signal spectrum and as long as the signal spectrum broadening does not impact neighbor channels. However, when chromatic dispersion is taken into consideration, the wider spectrum increases the temporal broadening.

One special case including chromatic dispersion and self-phase modulation can be derived from Eq. (12). If the second and third terms on the RHS of Eq. (12) cancel each other except for a phase shift from propagation, the signal only experiences a relative phase shift and fiber attenuation. This regime, which is known as the Soliton regime, requires the signal pulse to have a specific shape[4]:

$$A_{soliton}(z,T) = \sqrt{P_0}\,\text{sech}\left(\frac{T}{T_0}\right) \cdot \exp\left(j\frac{\gamma P_0}{2} z\right) \quad \text{with} \quad T_0 = \sqrt{\frac{-\beta_2}{\gamma P_0}} \tag{13}$$

The soliton pulse width, $T_0$, depends on the ratio of chromatic dispersion and pulse peak power. Therefore, to maintain a constant pulse shape, frequent signal amplification is required to maintain a constant signal amplitude despite the fiber attenuation. Alternatively, the dispersion of the fiber could be designed to decrease with propagation distance such that the reduced self-phase modulation with decreasing signal power is met by decreased fiber dispersion[5].

### 3.2 Cross-phase modulation

In WDM transmission, the phase of the signal in one channel is also modulated by the power in the other channels. Due to chromatic dispersion, the relative alignment of the signal pulses in different channels changes during propagation. This “walk-off” effect leads to an average phase modulation in the affected channel. There are two main effects resulting from cross-phase modulation: If the leading edge of a pulse starts colliding with a pulse in the affected channel, the pulse frequency in this channel is shifted. After a full walk-through of the pulses, the pulse frequency is shifted back – as long as the interaction length is short enough to neglect signal attenuation. During the walk-through, the frequency-shifted pulse experiences a different propagation velocity due to chromatic dispersion. For a full pulse walk through, the resulting pulse timing jitter can be approximated as:

$$\Delta\tau \approx \frac{\gamma \cdot E_2}{4\pi^2 \Delta f^2 \cdot \beta_2}, \tag{14}$$

where $E_2$ is the energy of the pulse in the neighbor channel and $\Delta f$ is the frequency separation between the channels. It can be seen that the larger the chromatic fiber dispersion $\beta_2$ the smaller the induced timing jitter. For example, in a 10 Gb/s system on non-zero dispersion shifted fiber (NZDSF), with a channel separation of 50 GHz and RZ pulses with 50 ps pulse width and 8 mW peak power (average signal power: 3 dBm), each pulse crossing with a neighbor channel pulse induces a timing shift of approximately 1 ps.

The second effect from cross-phase modulation distorts the signal shape: If the edge of a neighbor channel pulse coincides with a signal pulse in the affected channel at the input to a fiber span, the walk-off will induce a frequency shift in only part of the signal pulse. Subsequent chromatic dispersion converts this frequency modulation into amplitude distortion[6]. This effect increases the dispersion compensation requirements in the transmission system.

### 3.3 Four-wave mixing

The beating between different channels of the WDM system results in a power variation at the frequency of the channel separation. The term $|A|^2$ in Eq. (12) changes periodically with time and modulates the signal channel, generating modulation side bands, separated by the original channel spacing. This four-wave mixing effect is named after the two waves generating the modulation, the modulated wave and the generated side band. Two of these waves can coincide, such that only three waves contribute to the effect. If the generated wave coincides in frequency with another channel of the WDM system, this channel will be distorted. The multiple channel combinations in a WDM system make the distortion noise-like. To keep the impact of FWM below a penalty of about 1 dB, the system parameters should meet the requirement[7]

$$\frac{\gamma \cdot P_{avg}\sqrt{N_{spans}}}{|D| \cdot \Delta f^2} \le 7 \cdot 10^{-4}, \tag{15}$$

where $\gamma$ in 1/(W km), $P_{avg}$ is the average signal power per channel in mW, $N_{spans}$ is the number of amplified spans, D in ps/(nm km) and $\Delta f$ is the channel spacing in GHz. As an example, a 10 Gb/s system on NZDSF using NRZ signals with an average power of 2 mW and 50 GHz channel spacing would start experiencing problems from FWM after about 6 spans. Like with XPM, higher chromatic fiber dispersion and wider channel spacing alleviate the non-linear impact of the neighbor channels.

### 3.4 Intra-channel four-wave mixing

While FWM in medium to high rate transmission systems couples the signals in different channels, for very high rates (40 Gb/s and above) FWM can become an intra-channel effect and therefore be seen even in single-channel systems[8-10]. Due to chromatic dispersion, the signal pulses spread in time, overlapping the time slots of the neighbor pulses[8]. However, only part of the signal spectrum of each pulse overlaps with a different part of the neighbor pulse spectrum. The different frequency components in each time slot beat with each other, generating new modulation sidebands or FWM products, as schematically shown in the time-frequency picture in Figure 1. Dispersion compensation reshapes the signal pulses and four-wave mixing products, resulting in “shadow pulses” before and after the signal pulses.

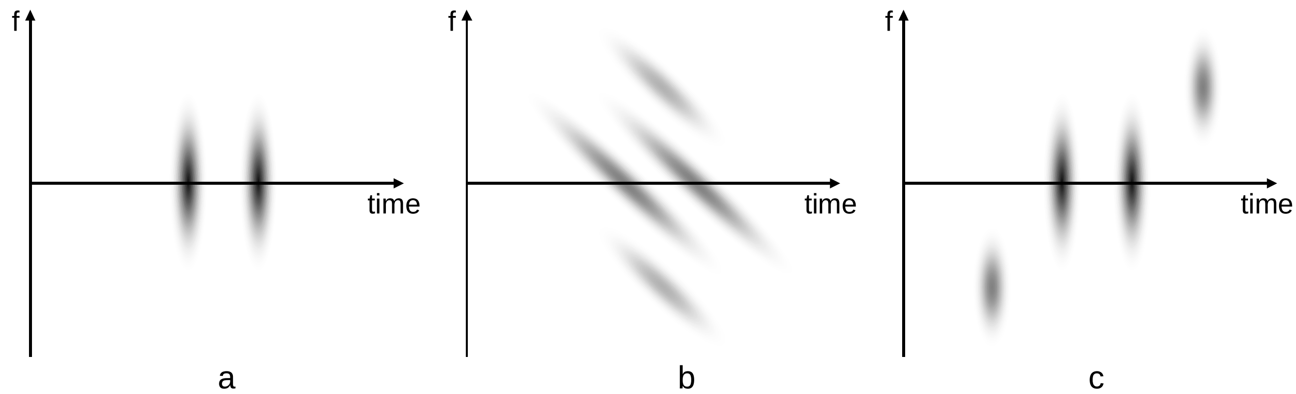


Figure 1: Schematic of intra-channel four-wave mixing: Signal intensity in frequency-time plane. a) Two pulses at fiber input. b) After propagation: Pulses have been chirped by chromatic dispersion, and two four-wave mixing sidebands have been generated. c) After dispersion compensation: Time shapes of signal pulses and four wave-mixing sidebands have been restored, leading to shadow pulses before and after the signal pulse pair.

## 4. SCATTERING EFFECTS

Two important non-linear effects are not based on the Kerr effect. They rather result in scattering of the signal wave by vibrational or acoustical waves in the fiber medium.

### 4.1 Stimulated Brillouin Scattering

An acoustic wave traveling counter-directionally to the signal wave will cause a refractive index grating in the fiber. The signal wave is then partly reflected (scattered) and downshifted in frequency by this traveling refractive index grating. The frequency shift is on the order of 11 GHz. In turn, the counter-propagating scattered wave interacts with the forward traveling wave, enhancing the acoustic wave. While this process is initiated by the interaction of noise and signal, it increases rapidly when the signal power is high enough, resulting in most of the signal power being backscattered. The threshold for the generation of a stable acoustic wave is on the order of 1 to 10 mW, depending on the fiber parameters. However, the bandwidth of the refractive index grating is very small, about 10 MHz. In order to reduce the effect of Brillouin scattering, it is enough to maintain sufficiently low power within each spectral slice of 10 MHz. This can be done by spectral broadening measures like choosing an appropriate modulation format like RZ modulation or by phase modulation of the signal carrier.

### 4.2 Stimulated Raman Scattering

The optical wave is also scattered by molecular vibrations in the fiber. The scattered wave is then downshifted in frequency, losing power to the molecular vibration. This effect is, like Brillouin scattering, initiated by an interaction between signal and noise, and can lead to depletion of the signal power if the initial optical power is sufficiently high. As the typical Raman threshold is on the order of nearly one Watt, Raman scattering is not likely to occur, when started from noise. If, however, another optical wave of sufficient power is present at the downshifted frequency, this wave interacts with the first wave, generating a strong molecular vibration. In turn, the first wave is scattered, amplifying the second optical wave. The downshift frequency can be between 0 and 15 THz, where the stronger interaction occurs for larger frequency separations, following an approximately triangular profile[11], as shown in Figure 2. The result is that the lower frequency wave ("signal wave") is amplified, while the higher frequency wave ("pump wave") loses power during this process. The pump and signal waves can travel in the same direction or in opposite directions in the fiber, yielding about the same effect. This Raman scattering is used for signal amplification. The Raman gain can be well approximated by

$$G(\Delta f) = \exp\left(\frac{1}{2}\frac{g_R(\Delta f)}{A_{eff}} \cdot P_{pump} \cdot L_{eff}\right). \qquad (16)$$

$g_R$ is the Raman gain coefficient, which depends on the frequency separation, $\Delta f$, between pump and signal, $A_{eff}$ is the fiber effective area, $P_{pump}$ is the power of the higher frequency pump wave at the fiber input, and $L_{eff}$ is the effective length of the fiber (see Eq. (4)) at the pump wavelength. The factor of ½ takes into account randomly varying relative states of polarization between the two waves. The Raman gain coefficient depends on the fiber effective area as well as the fiber Germanium doping. A typical value for a pump wavelength around 1500 nm is 6.5e-14 m/W with slightly larger values for higher doped fibers. For SSMF with an effective area of $80\mu m^2$ and an effective length of about 15 km for wavelengths between 1450 nm and 1500 nm, the Raman effect yields a gain of about 25 dB/W for wavelength separation of 100 nm, corresponding to about 13 THz. This Raman gain is approximately the same for co- and counter-propagating waves.

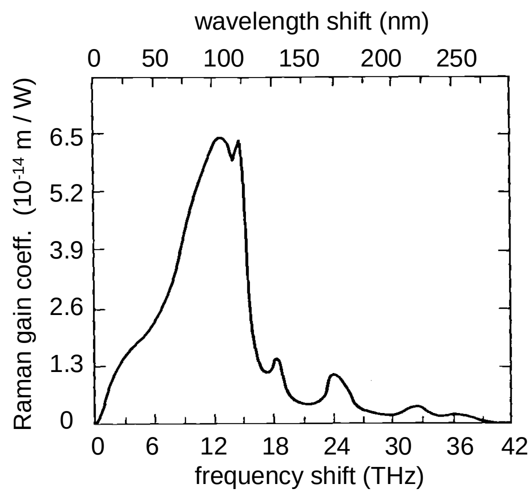


Figure 2: Measured Raman gain coefficient as function of frequency separation. Pump wavelength is 1550 nm. (Based on Ref. 12)

Usually, the Raman gain profile from Figure 2 is approximated by a triangular function[11] between frequency separation of 0 to 13 THz, corresponding to a wavelength separation of 0 to 100 nm.

While Raman scattering can be used as a beneficial effect to amplify signals in the transmission fiber, it also can introduce two types of signal distortion: In a WDM system, each signal acts as pump to amplify lower frequency signals. While the lower frequency signal gains power, the higher frequency signal is attenuated, resulting in a power tilt after propagation over a fiber span. If the Raman gain spectrum is approximated by a triangular function, the induced power tilt is linear (in decibel terms)[13]. In addition to the fiber Raman gain coefficient and the effective length of the span, the effective tilt only depends on the overall power in all channels and the overall optical bandwidth of the WDM system. The introduced tilt per fiber span, defined as the gain difference between lowest and highest frequency channel, can be approximated by

$$\Delta G = \exp\left(\frac{1}{2}\frac{g_{R,\max}}{A_{eff}} L_{eff} \cdot \frac{f_{\max} - f_{\min}}{13\,THz} \sum_i P_i\right) \qquad (17)$$

$P_i$ is the average power in channel i, and the summation is done over all channels of the WDM system. For the triangular gain spectrum approximation to be valid, the overall system bandwidth needs to be smaller than 13 THz. An alternate interpretation for the power tilt approximation is that each channel is amplified by the equivalent of a single pump, carrying the sum power of all channels and located at the power-based center of gravity of the channels. With a peak Raman gain in SSMF of about 35 dB/W for a pump around 1550 nm, a 1 dB gain difference between low and high frequency channels is introduced if the product of overall power and overall bandwidth is about 370 mW-THz, which is already met in a system with 40 channels, spaced by 100 GHz, and 2.5 mW per channel.

The effect of Raman scattering in this case is a higher effective fiber loss for the shorter wavelength channels, resulting in more required gain and lower OSNR for these channels. An additional distortion comes into play when the temporal variation of the channels due to data modulation is considered. The Raman gain that a channel receives from an pump channel is proportional to the time integral of the signal power in the pump channel over a time that corresponds to the walk-off time between the channels. An important aspect to evaluate the system impact of the randomly-varying Raman scattering is the standard deviation of the Raman gain between the two channels:

$$\sigma_1 = \frac{1}{2}\frac{g_{R,\max}}{A_{eff}} L_{eff} \frac{|f_2 - f_1|}{13\,THz} P_2 \tanh^{1/2}\left(\frac{\alpha T}{2|D\,\Delta\lambda|}\right), \qquad (18)$$

where $f_2 - f_1$ is the frequency separation between the two channels, $P_2$ is the average power in channel 2, T is the bit period in channel 2, D is the chromatic dispersion of the fiber, and $\Delta\lambda$ is the wavelength separation between the channels. For multiple channels, the gain variance of a channel is obtained by adding the variances incurred from all channels. If the power is equal in all $N_{ch}$ channels, and the channel spacing is constant at $\Delta f$, the resulting standard deviation of the power variation in channel $i$ can be approximated as

$$\sigma_i = \frac{1}{2}\frac{g_{R,\max}}{A_{eff}} L_{eff} \frac{\Delta f}{13\,THz} \sqrt{\frac{\alpha T}{2|D\Delta\lambda|}} N_{ch} \cdot P_{channel} \sqrt{f(i, N_{ch})}$$

$$\text{with } f(i, N_{ch}) \approx \frac{1}{4} + \left(\frac{i}{N_{ch}} - \frac{1}{2}\right)^2 \qquad (19)$$

The impact from Raman scattering is about twice as large in the channels on the edge of the transmission spectrum than in the center. Eq. (19) also demonstrates that higher chromatic dispersion *D* reduces the impact of Raman scattering due to better statistical averaging during pulse walk-off. It is also noteworthy that lower bit rates with larger bit duration T suffer more from Raman cross-talk. As an example, for a system at 10 Gb/s with 100 GHz channel spacing on non-zero dispersion shifted fiber (D=4 ps/(nm-km), 50 dB/W Raman gain), the first square-root term in Eq. (19) is about 0.88. If a standard deviation $\sigma_i$=0.05 yields a penalty of about 1 dB, then the overall signal power in the system is limited to 900 mW, if only a single span is considered. For a system over multiple amplified spans, the acquired variances are added up, such that for a system over 16 spans an overall power of only 230 mW is allowed for a 1 dB penalty from Raman cross-talk.

## 5. SUMMARY

We have discussed the linear and non-linear fiber effects and their impact on high data-rate WDM transmission. While some effects like chromatic dispersion, self-phase modulation and cross-phase modulation can be partly mitigated by appropriate dispersion compensation, other effects cannot be compensated in the optical domain. We have shown that most non-linear effects are reduced for larger chromatic dispersion in the fiber. On the other hand, of course, larger fiber dispersion requires a larger amount of dispersion compensation, which impacts the cost and the OSNR budgets of a transmission system. In any case, it is important to consider non-linear effects in the design of an optical transmission system.

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