Invited Paper

Short pulse self-focusing

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ABSTRACT

A short pulse will develop temporal sub-structures as it self-focuses. We show how these substructures lead to both the catastrophic expansion of the spectrum and the remarkable beam stability against self-focusing.

1. INTRODUCTION

Although self-focusing has been observed and investigated for over 20 years, it has been studied almost exclusively in the long pulse regime. In this limit, the paraxial ray approximation predicts that the beam will self-focus to moving singularities in the absence of other nonlinear effects. This model of self-focusing (called the moving focus model) is approximately valid for nanosecond pulses.

Major modifications are required to account for the experimental observation that ultrashort pulses resist self-focusing. Although short pulse resistance to self-focusing has never been explicitly noted, two well known observations, associated with continuum generation, conclusively demonstrate that short pulses stop self-focusing before the divergence of the beam has been severely altered:

(i) Continuum generation only occurs for powers exceeding the critical power for self-focusing, yet, the spatial properties of the continuum radiation are only slightly modified from the input beam.[1,2,3] This observation is inconsistent with a spatial instability leading to catastrophic beam collapse.

(ii) Ionization is absent from continuum generation experiments in extended media, even those in which the focused intensity in the absence of self-focusing is only slightly below the multiphoton ionization threshold. The lack of ionization is unambiguously demonstrated by the almost complete energy transmission through the focus in sub-100 fs pulse continuum experiments.[4] Thus, the beam could not have collapsed to the point where ionization (a competing nonlinearity) would dominate the process.

A new model is needed to describe self-focusing in the short pulse regime. When considering short pulse propagation, group velocity dispersion must be taken into account. We show that in normally dispersive media, group velocity dispersion is responsible for halting the self-focusing action. By relating continuum generation in extended media, to self-focusing in the short pulse limit, we emphasize the ubiquitous nature of the resistance of short pulses to self-focusing. In extended media, continuum generation is, in fact, the signature that the short pulse limit of self-focusing has been reached.

Our work should be contrasted with other recent work on self-focusing, which shows that simultaneous spatial and temporal collapse is possible in anomalously dispersive media.[5] Under very specific conditions in anomalously dispersive media, propagation of pulses that do not change in space or time is possible. However, in the absence of high order nonlinearities this solution is unstable.

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2. MODEL

The time-dependent paraxial wave equation, in the presence of group-velocity dispersion is

$$2ik\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)E+\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}-k\frac{\partial^2 k}{\partial \omega^2}\frac{\partial^2 E}{\partial t^2}+2k^2\frac{n_2}{n_0}|E|^2 E=0 \qquad (1)$$

where the refractive index is $n = n_0 + n_2 E^2$ and v_g is the group velocity.[5] The fifth and sixth terms in the equation are the dispersion and the nonlinear term, respectively. The dispersion term dominates over the nonlinearity for sufficiently short pulses. As an example, the dispersion of a 6 fs pulse in fused silica will equal the nonlinearity at an intensity of 10^{13} W/cm^2. Dispersion will also dominate for a 10 fs pulse in xenon gas at intensities less than 10^{13} W/cm^2. This will be true at any pressure, since both the dispersion and the nonlinearity are linearly dependent on pressure. At these pulse durations, a pulse will be stretched by dispersion before the divergence of the beam has been altered by the nonlinearity. Self-focusing in the extreme short pulse limit is then stopped before it begins.

Pulses of longer duration begin to self-focus as described by the moving focus model (slowly varying envelope approximation).[6,7,8] Because different segments of the pulse have different power, they follow different paths leading towards different focal positions with the highest power portion of the pulse having its focal position nearest to the lens. The consequence can be seen qualitatively. For each $\pi/2$ phase change produced by the nonlinearity, a distinct focus is created. With each new focal position that is created, the pulse is further segmented. For short pulses, there is a maximum phase retardation allowed before the slowly varying envelope approximation is violated. In the short pulse limit, the above critical power duration of the pulse determines the total phase retardation that can be imposed by self-focusing, regardless of the peak power. In the long pulse limit, competing nonlinearities will always occur and stop the self-focusing before the paraxial ray approximation is violated. With short pulses, dispersion stops the process before the slowly varying envelope approximation breaks down.

It is important to consider the interplay between the spatial (self-focusing) and the spectral (self-phase modulation) development. An on axis phase retardation of $\pi/2$ gives an associated spectral broadening of

$$\delta\omega=\frac{d\delta\varphi}{dt}=\frac{\pi}{2\tau}\approx\Delta\omega_{\text{init}} \qquad (2)$$

where τ and $\Delta\omega_{init}$ are the original pulse duration and bandwidth respectively. The added bandwidth is then approximately equal to the original bandwidth for a $\pi/2$ phase retardation. As the added spatial and spectral frequencies are rearranged by the combined actions of the nonlinearity and diffraction the shorter duration intensity structure is formed. Thus as the pulse propagates, the intensity profile of the pulse must change with a high intensity spike (caused by a radial contraction of the beam) forming on axis around the maximum power segment of the pulse.[9,10] Once shorter duration structures develop, Eq. 2 indicates that the same retardation produces a larger change in the bandwidth. As more bandwidth is produced the structures become shorter. Pulsed self-focusing is both a spatial and spectral instability. Much less phase modulation is required to produce a given bandwidth with the combined actions of self-phase modulation and self-focusing than with plane wave self-phase modulation alone. Since it takes little phase modulation to reach the dispersion limit, the beam undergoes minimal spatial distortion.

All parts of the pulse with power greater than the critical power undergo this process. As the pulse propagates towards the low power focus, the intensity spike travels forward and backward through the pulse from the highest power segment until the power falls below the critical level. The emergence of these temporal microstructures is an inevitable consequence of a pulse self-focusing.

The transmitted beam then radiates from a line focus. The line focus accounts[11] for the conical emission seen in self-focusing experiments.[3,12,13] The divergence of the beam at each focal position is determined by the total phase retardation. It is a function of the duration of the pulse that is above critical power. The positon of each focus and, therefore, the overall length of the line focus is a function of the pulse power. The spatial properties of a self-focused beam is a function of both the divergence and the length of the line focus.

Short pulse self-focusing in normally dispersive media is inextricably linked with continuum generation. The continuum spectrum is the result of the summation (and interaction) of the broadband radiation of the transient substructures. The frequency does not continuously vary throughout the pulse as would be the case for self-phase modulation alone, but rather the entire continuum spectrum is produced at each point in the pulse, with a definite phase relationship between each point determined by the original phase of the pulse. The spatial and spectral modulation, characteristic of the continuum spectra, results from the interference between the multiple broadbandwidth sources. Studies of the details of continuum radiation provides experimental access to the self-focusing process.

A complete, quantitative understanding of how the pulse evolves substructures and produces continuum will require a full theoretical treatment. We make no attempt to predict the continuum spectrum. The spectrum, produced by self-phase modulation of the temporal spikes, depends on their temporal profiles. Processes such as self-steepening would then need to be included in a complete model. Also since dispersion causes the extreme frequencies to move away from the spike, nonlinear frequency mixing between frequencies produced at different segments of the pulse must be included. Such a treatment is beyond the scope of this paper. Rather, we introduce a simplified model which contains enough of the essential characteristics of self-focusing to predict the spectral modulation observed in continuum spectra. A previous study has pointed out that the spectral modulation could not be understood using existing theories of continuum generation.[14]

The model replaces the transient substructures by an appropriately phased line array of sources. Since we are concerned with the spectral modulation rather than the overall spectrum, we can make the following simplifying assumptions:

(i) The pulse is Gaussian in time with a full width at half maximum duration of 50 fs. The duration of the pulse above critical power is divided into 2N segments.
(ii) Each segment of the pulse (labeled n) produces a white spectrum.
(iii) Each spectral component in segment n has the initial phase and amplitude of the incident pulse at retarded time, t_n. This is equivalent to the statement that all frequency components in all segments have the same phase retardation, which is a good approximation for small phase retardations. Note that the radiation measured along the optical axis will then be independent of the positions of the array elements.

The electric field at frequency, ω, produced by the phased sources at time, t, and a point z along the optical axis is then given by:

$$E(t,z,\omega) = \sum_{n=-N}^{N} E_n(t_n)\cos\left[\phi_n(t,z,\omega)\right] \tag{3}$$

where the phase φ_n is given by:

$$\phi_n(t,z,\omega) = (\omega_0 - \omega)t_n + \omega\left(t - \frac{z}{c}\right) \tag{4}$$

where E_n is the electric field strength of the the n^{th} segment of the pulse, ω_0 is the laser frequency, c is the speed of light. The resulting spectral intensity patterns are presented together with the experimental results.

3. EXPERIMENT

An experiment was performed to investigate the model predictions. Pulses were generated by amplifying the output of a colliding pulse modelocked (CPM) dye laser in a Nd:YAG pumped prism dye amplifier. The pulse energy was ~500 µJ and the duration was 50 fs (as measured on a multiple pulse autocorrelator). A vacuum spatial filter, containing an aperture of diameter less than the diffraction limit of the incident beam, produced an Airy pattern and the central maximum was selected with an iris. The resulting diffraction-limited beam was focused by a F/150 lens into a 75 cm long gas cell which could be evacuated or filled with rare gases to a pressure of up to 40 atmospheres.

An aperture at the output of the high pressure cell selected the radiation along the axis. The spectra were detected by an optical multichannel analyser at the output of a spectrometer. As our model is valid for the spectral modulation rather than the overall spectrum, coloured glass filters were placed in the beam to give an approximately uniform signal across the detector. The energy of the input pulse remained constant and the nonlinearity was varied by changing the gas pressure.

The figure shows the spectral modulation of continuum radiation for pressures of 13 and 21 atm. Continuum was observed for a minimum pressure of 5 atm. This observation establishes the self-focusing threshold.[3,12] The pressures then correspond to power thresholds of 0.38 and 0.24 of the peak power, respectively. The modulation period decreases with increasing pressure as is expected. As the threshold for self-focusing decreases, more of the pulse has the required power to self-focus and so the short structures are formed over a longer duration, resulting in faster modulation. The figure also shows calculated spectra for a 50 fs pulse, with power thresholds of 0.39 and 0.24 of the peak power, respectively. The figure shows that spectral modulation is a result of the transient structures produced during self-focusing.

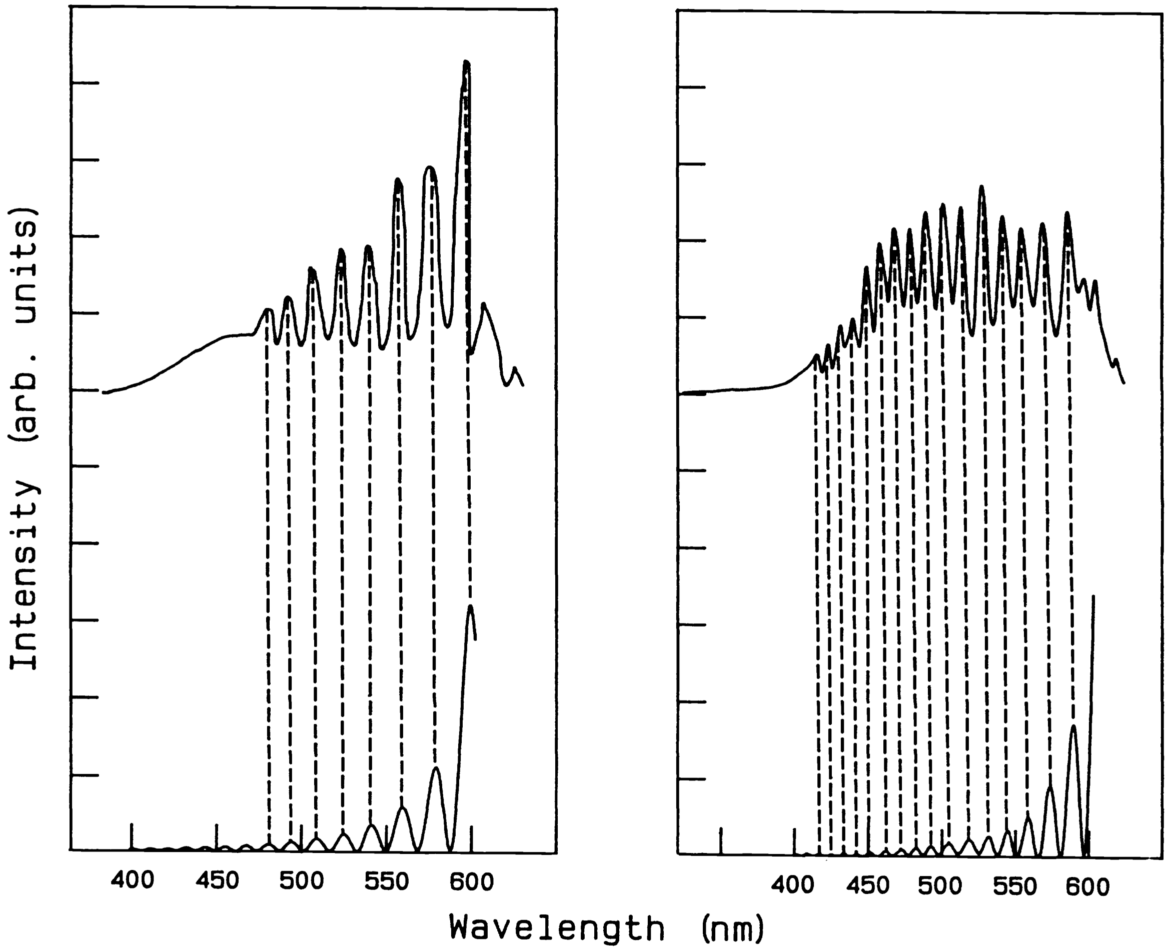

Figure. The top traces show experimental single-shot spectra. The cell pressure for the left trace was 13 atm; for the right, 21. The threshold pressure for continuum generation was 5 atm. The lower traces show the corresponding calculated spectra.

Each spectrum shown is produced by a single laser shot. The modulations are washed out when averaged over several shots. The modulation varies as the pulse duration and energy fluctuates. Also the modulations are sometimes irregular, presumably because of variations in the symmetry of the pulses. Asymmetric pulses will not give a regular modulation pattern.

4. CONCLUSION

In conclusion, we have introduced and experimentally confirmed one prediction of a model of short pulse self-focusing which indicates qualitatively different behaviour from long pulse self-focusing, including a remarkable beam stability. This behaviour will be characterisitic of all nonlinear media, whether solid, liquid or gas, in the regular dispersion region. Much research since the introduction of the moving focus model has concentrated on finding tractable, realistic models of self-focusing that do not show spatially singular behaviour. Singular behaviour will not occur for short pulse self-focusing. With the basic physics identified, it is now possible to develop a complete description of short pulse self-focusing.

5. REFERENCES

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