Light Emission Problem in Classical and Quantum Optics

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Abstract: Understanding of light emission is a keystone in mastering classical and quantum optics. We review and compare approaches used in these two fields for description of light emission to point out and clarify common misconceptions.

1. Introduction

Understanding of light emission takes a key part in mastering classical and quantum optics [1, 2]. In the following sections, we review and compare approaches used in classical (wave) and quantum descriptions of light radiation. In particular, we consider magnetic multipole radiation emitted by a monochromatic magnetization current with the density

$$\boldsymbol{J}(t,\boldsymbol{r}) = \nabla \times \boldsymbol{M}(\boldsymbol{r})\, e^{-i\omega t}, \tag{1}$$

where $\boldsymbol{M}(\boldsymbol{r})$ is an arbitrary magnetization field, and ω is the angular frequency of the current. The density $\boldsymbol{J}(t,\boldsymbol{r})$ is the elementary current density that constitutes any magnetization current contributed by either induced or intrinsic magnetic moments of particles [3, 4]. We take this current as an example in our review because of its pure transverse fields that can be completely described with vector potential, enabling clear comparison of the two optical approaches.

2. Classical description

Classical description of magnetic multipole radiation is straightforward and mathematically exact. It is based on Maxwell's equations, which can be reduced to the following one [1]:

$$\boldsymbol{\nabla}^2\boldsymbol{A}(t,\boldsymbol{r}) + k_0^2\boldsymbol{A}(t,\boldsymbol{r}) = -\mu_0\,\boldsymbol{J}(t,\boldsymbol{r}), \tag{2}$$

for the vector potential $\boldsymbol{A}(t,\boldsymbol{r})$ in the transverse gauge $\boldsymbol{\nabla}\cdot\boldsymbol{A}(t,\boldsymbol{r}) = 0$. In this equation, $k_0 = |\omega|/c$ is the vacuum wavenumber with the speed of light in free space $c = (\varepsilon_0\mu_0)^{-1/2}$ and electric and magnetic constants ε_0 and μ_0.

In the absence of background fields, Eq. (2) has the following exact solution [1, 4]:

$$\boldsymbol{A}(t,\boldsymbol{r}) = \boldsymbol{A}_{DF}(t,\boldsymbol{r}) = \mu_0\boldsymbol{\nabla}\times\int_0^\infty \frac{\exp[i(k_0|\boldsymbol{r}-\boldsymbol{r}'|-\omega t)]}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\boldsymbol{M}(\boldsymbol{r}')d\boldsymbol{r}'. \tag{3}$$

$\boldsymbol{A}_{DF}(t,\boldsymbol{r})$ is the field of pure or so-called dark-field magnetic multipole radiation. If we decompose the magnetization field $\boldsymbol{M}(\boldsymbol{r}) = \int\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{M}(\boldsymbol{r}')d\boldsymbol{r}'$ over continuously distributed magnetic dipoles with local strength $\boldsymbol{M}(\boldsymbol{r}')d\boldsymbol{r}'$, we can write the elementary field radiated by a single magnetic dipole as follows:

$$d\boldsymbol{A}_{DF}(t,\boldsymbol{r}) = \mu_0\boldsymbol{\nabla}\times\frac{\exp[i(k_0|\boldsymbol{r}-\boldsymbol{r}'|-\omega t)]}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\boldsymbol{M}(\boldsymbol{r}')d\boldsymbol{r}'. \tag{4}$$

This is the classical magnetic dipole field [4] that decays as $\sim|\boldsymbol{r}-\boldsymbol{r}'|^{-2}$ in the near-field zone ($k_0|\boldsymbol{r}-\boldsymbol{r}'| \ll 1$) and transforms into a transverse plane wave $\sim\exp[i(k_0 r-\omega t)]$ in the far-field zone ($k_0|\boldsymbol{r}-\boldsymbol{r}'| \gg 1$). Superimposing fields (4) from continuously distributed in space magnetic dipoles of varying strength, we get multipole fields of very complex structure at the location of the current and superposition of plane-waves in the far-field zone common for all the dipoles.

In solution (3), we assumed that there is no background field at the location of the current. In a more complex, so-called bright-field, configuration, we have additional external fields $\boldsymbol{A}_{ext}(t,\boldsymbol{r})$ generated by currents $\boldsymbol{J}_{ext}(t,\boldsymbol{r})$ located outside of the considered system. In this case, Eq. (2) has more general solution [1],

$$\boldsymbol{A}(t,\boldsymbol{r}) = \boldsymbol{A}_{BF}(t,\boldsymbol{r}) = \boldsymbol{A}_{DF}(t,\boldsymbol{r}) + \boldsymbol{A}_{ext}(t,\boldsymbol{r}), \tag{5}$$

where the external field is given by superposition

$$\boldsymbol{A}_{ext}(t,\boldsymbol{r}) = \sum_{\alpha=1}^{2}\int_0^{4\pi}\boldsymbol{A}_{\alpha\boldsymbol{n}}(t,\boldsymbol{r})\,d\Omega_{\boldsymbol{n}} \tag{6}$$

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of elementary plane-wave free fields

$$\boldsymbol{A}_{\alpha\boldsymbol{n}}(t,\boldsymbol{r}) = \boldsymbol{A}^0_{\alpha\boldsymbol{n}} e^{i(k_0\boldsymbol{n}\cdot\boldsymbol{r}-\omega t)} \quad (7)$$

that propagate with orthogonal complex vector amplitudes $\boldsymbol{A}^0_{\alpha\boldsymbol{n}}$ in the direction specified by the unit vector $\boldsymbol{n}$ within the solid angle $d\Omega_{\boldsymbol{n}}$. The vector amplitudes of external field, $\boldsymbol{A}^0_{\alpha\boldsymbol{n}}$, are always orthogonal to $\boldsymbol{n}$, featuring $\boldsymbol{A}^0_{\alpha\boldsymbol{n}} \cdot \boldsymbol{n} = 0$ and $\boldsymbol{A}^0_{1\boldsymbol{n}} \cdot \boldsymbol{A}^0_{2\boldsymbol{n}} = 0$.

The bright-field magnetic multipole radiation is a very common model in classical optics. In particular, it is used to describe interaction of incident light with a magnetically active medium [3, 4]. In this case, $\boldsymbol{M}(\boldsymbol{r})$ is not the independent parameter, but the distribution of induced magnetic moments governed by the external field, $\boldsymbol{M}(\boldsymbol{r}) = \hat{\alpha}\, \boldsymbol{A}_{ext}(t,\boldsymbol{r})e^{i\omega t}$, where $\hat{\alpha}$ is the linear operator describing magnetizability of the medium at angular frequency ω. Within this model, the external field is the field of incident light that is scattered and absorbed by induced magnetic multipoles in the medium.

3. Quantum description

Quantum description of magnetic multipole radiation is somewhat cumbersome and mathematically less accurate as compared to classical treatment. It is based on solution of the Schrödinger equation with the interaction operator

$$\hat{V}_{int}(t) = \int_0^\infty \hat{\boldsymbol{J}}(t,\boldsymbol{r}) \cdot \hat{\boldsymbol{A}}(t,\boldsymbol{r})\, d\boldsymbol{r}, \quad (8)$$

where the current density and vector potential are operators [2]. The matrix elements of the current density operator $\boldsymbol{J}_{fi}(t,\boldsymbol{r}) =< f \big| \hat{\boldsymbol{J}}(t,\boldsymbol{r}) \big| i >$ contain only magnetic-type transitions of the emitter that exhibit $\boldsymbol{\nabla} \cdot \boldsymbol{J}_{fi}(t,\boldsymbol{r}) = 0$. The vector field operator $\hat{\boldsymbol{A}}(t,\boldsymbol{r})$ is given in the form of quantized free field of the transverse gauge,

$$\hat{\boldsymbol{A}}(t,\boldsymbol{r}) = \sum_{\alpha=1}^{2} \int_0^{4\pi} \hat{c}^+_{\alpha\boldsymbol{n}} \boldsymbol{A}^*_{\alpha\boldsymbol{n}}(t,\boldsymbol{r})\, d\Omega_{\boldsymbol{n}}, \quad (9)$$

where $\boldsymbol{A}_{\alpha\boldsymbol{n}}(t,\boldsymbol{r})$ is given by Eq. (7) with $\boldsymbol{A}^0_{\alpha\boldsymbol{n}}$ normalized to one photon in the volume. The creation operator $\hat{c}^+_{\boldsymbol{n}}$ increases the number of photons $N_{\alpha\boldsymbol{n}}$ by one, possessing the matrix elements $< N_{\alpha\boldsymbol{n}}|\hat{c}^+_{\alpha\boldsymbol{n}}|N_{\alpha\boldsymbol{n}} - 1 >= \sqrt{N_{\alpha\boldsymbol{n}}}$ only.

The exact solution of the Schrödinger equation with $\hat{V}_{int}(t)$ cannot be obtained. However, it can be solved within the perturbation theory [2], assuming that $\hat{V}_{int}(t)$ gives negligibly small contribution to the Hamiltonian operator. With this calculation, we can get lower-order estimations for the probabilities of single photon emission in direction $\boldsymbol{n}$ for all possible magnetic-type state transitions of the emitter.

4. Conclusion

Direct comparison of classical and quantum treatments of magnetic multipole radiation clearly demonstrates their similarities and differences. First of all, quantum description with an initial field state containing at least one photon corresponds to classical bright-field radiation, where field operator (9) is the quantum analog of classical external field (6). However, in contrast to classical solution, where the total field is sought as superposition of the forced $\boldsymbol{A}_{DF}(t,\boldsymbol{r})$ and free $\boldsymbol{A}_{ext}(t,\boldsymbol{r})$ fields, the quantum solution gives it in the form of free fields only. Mathematically, forced and free fields are linearly independent solutions that cannot be treated equally [5]. This brings up the fundamental question of completeness of quantum optics apparatus, as it was developed strictly for free fields [2].

Going back to our comparison of the two optical approaches, we conclude that the analog of classical near fields of $\boldsymbol{A}_{DF}(t,\boldsymbol{r})$ are ignored in quantum description, while the respective far fields of $\boldsymbol{A}_{DF}(t,\boldsymbol{r})$ are accounted through increase of free-field quanta in the final state. Thus, quantum description uses the far-field approximation for the radiated field which is valid strictly in the far-field zone. This contributes the radiated-field error to $\hat{V}_{int}(t)$. The lower the number of photons in the initial field state, the higher the relative error in $\hat{V}_{int}(t)$. Finally, if the field consists of zero photons in the initial state, then the operator $\hat{V}_{int}(t)$ gets significant errors as its calculation becomes fully given by the approximated radiated field with no contribution of exact incident fields. Thus, mathematically accurate description of pure radiation in the dark-field case is impossible in quantum optics, at least in terms of free-field quanta.

5. References

[1] L. D. Landau and E. M. Lifshitz, *The classical theory of fields*, 2nd edn. (Pergamon Press, 1962).
[2] V. B. Berestetskii, L. P. Pitaevskii and E. M. Lifshitz, *Quantum electrodynamics*, 2nd edn. (Elsevier, 2012).
[3] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd edn. (Pergamon Press, 1984).
[4] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, 1941).
[5] Y. Akimov and P. Rutkevych, "Photon model of light: Revision of applicability limits," *arXiv*:2106.08140 [physics.gen-ph].