Quantum Dense Coding with N-Participants in N Dimensions

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ABSTRACT

Quantum dense coding is an important protocol in quantum communication, and it utilizes the quantum entanglement to increase the capacity of quantum communication. This article introduces the basic protocol of quantum dense coding, and also extends the concept of quantum dense coding to higher dimensions and with more participants. The security of quantum dense coding in those conditions is also discussed; and the discussion shows the potential of quantum dense coding in various applications.

Keywords: quantum dense coding, quantum communication, security

1. INTRODUCTION TO QUANTUM DENSE CODING

The development of quantum information technology provides a more efficient and secure way of communication. Quantum dense coding, one of the foundations of quantum information technology, allows information carriers to transmit more information than using traditional means. Over the years, researchers have made significant progress in understanding the theoretical foundations of quantum dense coding and exploring its practical implementations. The development in encoding schemes, error correction techniques, and quantum technologies lays a solid foundation for the experimental realization of dense coding protocols [1]. Experimental demonstrations have been conducted using various physical systems, including photons, trapped ions, and superconducting qubits [2-4]. These experiments validate the feasibility of quantum dense coding and prove its potential in real-world applications.

2. QUANTUM DENSE CODING IN VARIOUS CONDITIONS

To delve into the depths of quantum dense coding, it is essential to understand its foundations. Charles H. Bennett and Stephen J. Wiesner introduce the concept of quantum dense coding first in 1992, and by using the properties of quantum entangled pairs, the purpose of their protocol is to transmit more information than the traditional method for every EPR pairs transmitted [5, 6].

2.1 Bennett-Wiesner Scheme

Supposed Alice want to send 2 bits of information to Bob. The quantum dense coding allows Alice to pass two bits of information to Bob by sending one quantum bit. In the following example, the maximally entangled state (Bell state) is used [7].

Alice and Bob share a EPR pair:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

To preparing the quantum states, the Hadamard gate and the CNOT gate is used.

$$\mathrm{CNOT}(\mathrm{H} \otimes \mathrm{I})|00\rangle = |\phi^+\rangle$$

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The classic bit that Alice want to pass to Bob only have four possibilities, which are “00”, “01”, “10”, “11”. For which possibility, a unitary matrix U is used to operate on the particle that owned by Alice. The Alice needs to do $U \otimes I\,|\phi^+\rangle$ operation to her particle.

If Alice wants to pass cbit = 00 to Bob, after the operation she would get:

$$I \otimes I\,|\phi^+\rangle = |\phi^+\rangle$$

If Alice wants to pass cbit = 01 to Bob, after the operation she would get:

$$\sigma_x \otimes I|\phi^+\rangle = |\psi^+\rangle$$

If Alice wants to pass cbit = 10 to Bob, after the operation she would get:

$$\sigma_y \otimes I|\phi^+\rangle = |\phi^-\rangle$$

If Alice wants to pass cbit = 11 to Bob, after the operation she would get:

$$\sigma_z \otimes I|\phi^+\rangle = |\psi^-\rangle$$

After this operation, Alice could pass her particle to Bob, and Bob could apply a proper unitary operation (expressing by unitary matrix B) on the matrix to measure the particle and get the classic information. To obtain the classic information:

$$B = \left(\mathrm{CNOT}(H \otimes I)\right)^{-1} = (H \otimes I)\mathrm{CNOT}$$

$$B|\phi^+\rangle = |00\rangle$$

$$B|\psi^+\rangle = |01\rangle$$

$$B|\phi^-\rangle = |10\rangle$$

$$B|\psi^-\rangle = |11\rangle$$

2.2 Dense coding with three participants (two information sender and one receiver)

Traditionally, quantum dense coding has been studied in the context of two participants (Alice and Bob) using a maximally entangled state such as the Bell state. This article shows the possibility of extending the concept of quantum dense coding into high dimensions and with more participants. Assume both Alice and Bob want to deliver a message to Charlie using quantum dense coding. The entangled particle pair shared by them is:

$$|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

To prepare the quantum state, the Hadamard gate is used as the normal dense coding.

$$\mathrm{CNOT} \otimes \mathrm{CNOT}(H \otimes I)|000\rangle = |\phi^+\rangle_{ABC}$$

The classic information that Alice and Bob try to deliver could only be 0 or 1. A unitary matrix is needed to operate on the particle pair in order to deliver the classic information.

The unitary matrix U_A for Alice is defined as:

$$U_A: \{I_2, \sigma_x\} \to \{0,1\}$$

The unitary matrix U_B for Bob is defined as:

$$U_B: \{I_2, \sigma_x\} \to \{0,1\}$$

Therefore, when both Alice and Bob is delivering their information, the operation on the particle is:

$$(U_A \otimes U_B \otimes I_C)|\phi^+\rangle_{ABC}$$

To be more specific, the four conditions are:

$$(I_A \otimes I_B \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$(I_A \otimes \sigma_B \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{2}}(|010\rangle + |101\rangle)$$

$$(\sigma_A \otimes I_B \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{2}}(|100\rangle + |011\rangle)$$

$$(\sigma_A \otimes \sigma_B \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{2}}(|110\rangle + |001\rangle)$$

The four quantum states got from the operation are orthogonal to each other. After this operation, Alice and Bob need to give their particle to Bob, and Bob would apply a proper unitary matrix B to the particle in order to obtain the classic information.

$$B = \left(CNOT \otimes CNOT(H \otimes I)\right)^{-1} = (H \otimes I)CNOT \otimes CNOT$$

2.3 Considering the communication with more participants

Supposed that we have 4 participants, Alice, Bob, Charlie, and David, and Alice, Bob, and Charlie try to deliver a message to David using quantum dense coding.

The entangled particles shared by them is:

$$|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

The following operation is used for them to prepare the entangled particles.

$$CNOT \otimes CNOT \otimes CNOT(H \otimes I_4)|0000\rangle = |\psi\rangle_{ABC}$$

The classic information that Alice, Bob, and Charlie try to deliver could only be 0 or 1. A unitary matrix is needed to operate on the particle pair in order to deliver the classic information.

The unitary matrix U_A for Alice is defined as:

$$U_A: \{I_2, \sigma_x\} \rightarrow \{0,1\}$$

The unitary matrix U_B for Bob is defined as:

$$U_B: \{I_2, \sigma_x\} \rightarrow \{0,1\}$$

The unitary matrix U_C for Charlie is defined as:

$$U_C: \{I_2, \sigma_x\} \rightarrow \{0,1\}$$

Therefore, when all of them is delivering their information, the operation on the particle is:

$$(U_A \otimes U_B \otimes U_C \otimes I_D)|\psi\rangle_{ABCD}$$

To be more specific, the four conditions are:

$$(I_A \otimes I_B \otimes I_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

$$(I_A \otimes I_B \otimes \sigma_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|0010\rangle + |1101\rangle)$$

$$(I_A \otimes \sigma_B \otimes I_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|0100\rangle + |1011\rangle)$$

$$(\sigma_A \otimes I_B \otimes I_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|1000\rangle + |0111\rangle)$$

$$(I_A \otimes \sigma_B \otimes \sigma_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$

$$(\sigma_A \otimes I_B \otimes \sigma_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|1010\rangle + |0101\rangle)$$

$$(\sigma_A \otimes \sigma_B \otimes I_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|1100\rangle + |0011\rangle)$$

$$(\sigma_A \otimes \sigma_B \otimes \sigma_C \otimes I_D)|\psi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|1110\rangle + |0001\rangle)$$

2.4 Extend to n-qubit

For n-qubit, the quantum state that is shared by participants is shown below. The formula has n number of 0s and n number of 1s.

$$|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|0\ldots0\rangle + |1\ldots1\rangle)$$

To prepare the quantum state, the Hadamard gate is used as the normal dense coding.

$$\left(\bigotimes_{i=1}^{n-1} CNOT\right)(H \otimes I)|0\ldots0\rangle = |\psi\rangle_{ABC}$$

For each qubit, the unitary matrices used to transform the quantum state are defined as:

$$U_i\colon \{I_2, \sigma_x\} \rightarrow \{0,1\}, i \in [1, n]$$

Therefore, the operation for all n participants to deliver their information is:

$$\left(\bigotimes_{i=1}^{n} U_i\right)|\psi\rangle_{ABC}$$

After the recipient gets the information from n sender, the recipient could decipher the classic information by applying operation B onto the quantum state. The matrix B is defined as:

$$B = \left(CNOT^{n-1}(H \otimes I)\right)^{-1} = (H \otimes I)CNOT^{n-1}$$

3. EXTEND THE QUANTUM DENSE CODING WITH THREE PARTICIPANTS INTO THREE-DIMENSION

Alice, Bob, and Charlie need to prepare a entangle state, and they would possess the first, second, and third quantum bit respectively.

$$|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)$$

To prepare the quantum state, the 3-dimensional Hadamard gate is needed.

$$\mathrm{CNOT}(H_3 \otimes I)|00\rangle = |\phi^+\rangle$$

$$H_3 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{pmatrix}$$

Where $w = e^{i\frac{2\pi}{3}}$

The operation that Alice and Bob apply on the particles are defined as:

Alice's operation is $U_A: \{I_3, V_a, V_b\} \rightarrow \{0, 1, 2\}$, where $V_a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $V_b = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

Bob's operation is $U_B: \{I_3, V_a, V_b\} \rightarrow \{0, 1, 2\}$, where $V_a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $V_b = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

The basic operations are:

$$(I_A \otimes I_B \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)$$

$$(I_A \otimes V_a \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|020\rangle + |101\rangle + |212\rangle)$$

$$(I_A \otimes V_b \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|010\rangle + |121\rangle + |202\rangle)$$

$$(V_a \otimes I_B \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|200\rangle + |011\rangle + |122\rangle)$$

$$(V_b \otimes I_B \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|100\rangle + |211\rangle + |022\rangle)$$

$$(V_a \otimes V_a \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|220\rangle + |001\rangle + |112\rangle)$$

$$(V_a \otimes V_b \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|210\rangle + |021\rangle + |102\rangle)$$

$$(V_b \otimes V_a \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|120\rangle + |201\rangle + |012\rangle)$$

$$(V_b \otimes V_b \otimes I_C)|\phi^+\rangle_{ABC} = \frac{1}{\sqrt{3}}(|110\rangle + |221\rangle + |002\rangle)$$

By replacing 1 in matrix V_a and V_b with w and w^2, $w = e^{i\frac{2\pi}{3}}$, we could get other operations.

3.1 Extend the quantum communication into high-dimensional

Alice, Bob, and Charlie need to prepare a entangle state, and they would possess the first, second, and third quantum bit respectively.

$$|\psi\rangle_{ABC} = \frac{1}{2}(|000\rangle + |111\rangle + |222\rangle + |333\rangle)$$

To prepare the quantum state, the 3-dimensional Hadamard gate is needed.

$$\mathrm{CNOT} \otimes \mathrm{CNOT}(H_4 \otimes I)|000\rangle = |\psi\rangle$$

$$H_4 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

The operation that Alice and Bob apply on the particles are defined as:

Alice's operation is $U_A: \{I_3, V_a, V_b, V_c\} \rightarrow \{0,1,2,3\}$, where $V_a = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, $V_b = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, $V_c = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$.

Bob's operation is $U_B: \{I_3, V_a, V_b, V_c\} \rightarrow \{0,1,2,3\}$, where $V_a = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, $V_b = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, $V_c = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$.

The basic operations are:

$$(I_A \otimes I_B \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|000\rangle + |111\rangle + |222\rangle + |333\rangle)$$

$$(I_A \otimes V_a \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|030\rangle + |101\rangle + |212\rangle + |323\rangle)$$

$$(I_A \otimes V_b \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|010\rangle + |121\rangle + |232\rangle + |303\rangle)$$

$$(I_A \otimes V_c \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|020\rangle + |131\rangle + |202\rangle + |313\rangle)$$

$$(V_a \otimes I_B \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|300\rangle + |011\rangle + |122\rangle + |233\rangle)$$

$$(V_b \otimes I_B \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|100\rangle + |211\rangle + |322\rangle + |033\rangle)$$

$$(V_c \otimes I_B \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|200\rangle + |311\rangle + |022\rangle + |133\rangle)$$

$$(V_a \otimes V_a \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|330\rangle + |001\rangle + |112\rangle + |223\rangle)$$

$$(V_a \otimes V_b \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|310\rangle + |021\rangle + |132\rangle + |203\rangle)$$

$$(V_a \otimes V_c \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|320\rangle + |031\rangle + |102\rangle + |213\rangle)$$

$$(V_b \otimes V_a \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|130\rangle + |201\rangle + |312\rangle + |023\rangle)$$

$$(V_b \otimes V_b \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|110\rangle + |221\rangle + |332\rangle + |003\rangle)$$

$$(V_b \otimes V_c \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|120\rangle + |231\rangle + |302\rangle + |013\rangle)$$

$$(V_c \otimes V_a \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|230\rangle + |301\rangle + |112\rangle + |123\rangle)$$

$$(V_c \otimes V_b \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|210\rangle + |321\rangle + |232\rangle + |103\rangle)$$

$$(V_c \otimes V_c \otimes I_C)|\psi\rangle_{ABC} = \frac{1}{2}(|220\rangle + |331\rangle + |002\rangle + |113\rangle)$$

By replacing 1 in matrix V_a, V_b, and V_c with I, -1, -i, we could get other operations.

To measure the particles and decipher the information, Charlie could use the following operation:

$$B = \left(CNOT \otimes CNOT(H_4 \otimes I_3)\right)^{-1} = \left({H_4}^3 \otimes I_3\right)CNOT \otimes CNOT$$

4. THE SECURITY IN QUANTUM DENSE CODING

It is well known that the quantum teleportation is hard to be attacked by classic method. The security of quantum communication is generally based on the properties of the entangled quantum state and the No-cloning theorem.

The basic unit in quantum communication is qubit rather, and in classical teleportation, the basic unit is cbit. The most essential difference between qubit and cbit is that qubit represents a set of linear combinations but cbit could only represent very limited information (usually 0 and 1). Take the quantum dense coding as an example, in the previously introduced encoding, only two qubit is used, and each qubit could represent the linear combination of $|0\rangle$ and $|1\rangle$,which greatly increase the difficulty for the attackers to decipher the information.

To elaborate, an attacker has 50% of possibility guessing a classical particle correctly because the only possibility is 0 or 1. However, in the encoding introduced above, without help from the sender, the attacker only has 1/4 of the possibility to correctly gain the information [8].

Another important property of qubit is that it cannot be duplicated. In quantum communication, a Unitary matrix operation can be implemented with 100% accuracy. The No-cloning theorem for the qubit could be proved by contradiction.

Assume we have two arbitrary quantum states $|\psi\rangle$ and $|\phi\rangle$, and there exist a unitary operator U that could copy them onto another state $|k\rangle$ which is completely irrelevant to them. Thus, the copy operator U must satisfy the following equations.

$$U(|\phi\rangle \otimes |k\rangle) = |\phi\rangle \otimes |\phi\rangle$$

$$U(|\psi\rangle \otimes |k\rangle) = |\psi\rangle \otimes |\psi\rangle$$

By taking the inner product $\langle U(\phi \otimes \psi)|U(\psi \otimes \phi)\rangle$, the following equations could be derived.

$$\langle U(\phi \otimes \psi)|U(\psi \otimes \phi)\rangle = \langle \phi \otimes \phi|\, \psi \otimes \psi\rangle$$

$$\langle U(\phi \otimes \psi)|U(\psi \otimes \phi)\rangle = \langle \phi \otimes k|\, \psi \otimes k\rangle$$

Therefore,

$$\langle \phi \otimes \phi|\, \psi \otimes \psi\rangle = \langle \phi \otimes k|\, \psi \otimes k\rangle$$

$$\langle \phi|\, \psi\rangle\langle \phi|\, \psi\rangle = \langle \phi|\, \psi\rangle\langle k|\, k\rangle$$

Because of $\langle k|\, k\rangle = 1$,

$$\langle \phi|\psi\rangle^2 = \langle \phi|\psi\rangle$$

This means that the only solutions are $\langle \phi|\, \psi\rangle = 0$ or $\langle \phi|\, \psi\rangle = 1$, which means either $\phi = \psi$, or they are orthogonal to each other. This is contradicted with the assumption that ϕ and ψ are arbitrary.

The No-cloning theorem shows that the attacker cannot simply copy the qubit, which granted the security of quantum teleportation.

4.1 The Security of quantum dense coding for multi-party communication

In the communication protocols discussed above, it is crucial to discussed about how many information that David could decipher if David only gets a part of particles rather than all particles.

Considering the situation that David only get particles from Bob, and Charlie but not from Alice:

If Alice could not provide further help, Charlie, Bob could measure the particle that he has with the operation below:

$$B_3 = \left(CNOT^3(H \otimes I_3)\right)^{-1} = (H \otimes I_3)CNOT^3$$

By measured with matrix B_3, David could still get the information from Bob and Charlie, but he cannot get the information form Alice without having her particle.

If Alice could measure her particle and send the result to David, David could decipher all the original information.

Thinking about a more general example, supposed a receiver could only get n number of particles form m number of senders. The total number of combinations of information is 2^m. Without any other help, the receiver has $\frac{1}{2^{m-n}}$ chance of guessing the original information correctly. If the receiver gets p number of helps from the senders, the chance of guessing correctly is $\frac{1}{2^{m-n-p}}$. Therefore, in order to decipher all the classic information, the receiver needs helps from at least m-n helps from senders.

5. CONCLUSION

The implementation of quantum dense coding is based on the properties of entangled quantum pairs, and the quantum entanglement has been widely studied and experimented, which provides a solid foundation for the development of quantum teleportation [9]. Due to the security and the ability of transmitted information efficiently, quantum dense coding could be applied to various fields, including quantum cryptography and quantum computing [10]. Moreover, the demonstration of that quantum dense coding could be applied to high dimensions and to more participants shows quantum dense coding is technology that have potential and can still be developed.

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