Performance of Chirp Spread Spectrum in the Multipath Channel

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ABSTRACT

In wireless communication, fading caused by multipath interference is the dominant factor reduces the performance of the system. In this letter, the performance analysis of Chirp-BOK modulation in Rayleigh channel and is carried out and the bit error rate (BER) function is derived and the multipath interference that influence on coherent demodulation performance and signal energy are analyzed. The simulation result demonstrates the derivation is validity.

Keywords: Chirp modulation; multipath interference; bit error rate (BER).

1. INTRODUCTION

Chirp signal is also known as linear modulated frequency (LMF) signal, associated with technique of pulse compression have been extensively used in radar system [1], [2]. Chirp signal is representative non-stationary signal which instantaneous frequency changes over time developing rapidly [3]. The characteristics of low power consumption, high processing gain, and low cost make it's widely used in wireless communication as modulation signals [4]. Compared with traditional carrier signals, the data communication using chirp signals does not necessarily employ coding and produces a transmitted bandwidth much greater than the bandwidth of the information signal being transmitted [5]. Chirp modulation has inherent interference rejection capability, especially in the presence of Doppler shifts and fading due to multipath propagation [6], [7], [8]. In the literature, several studies have concentrated on the structure and modem scheme of Chirp modulation [9]. the BER performance of Chirp modulation in multipath channel is not available. The closed-form BER performance of Chirp modulation in multipath channel is quite significant in the communication system design and analysis.

The paper is organized as follows. Section ii introduces the theory of wideband signal anti multipath. In section iii, the system performance of Chirp-BOK modulation is analyzed in Rayleigh channel. The bit error rate (BER) function is derived in multipath interference in section iiii. The paper is concluded in section IIIii.

2. ANTI INTERFERENCE THEORY OF WIDEBAND SIGNAL

According to Shannon's hypothesis signals with white noise statistical characteristics in Gaussian Noise (AWGN) channel are the optimal signal form for transmission [10]. In the case of dual waveform transmission, the two signals received by the receiver are $s(t)$ and $s(t+\tau)$ respectively. Therefore, the synthesized signal power is

$$\begin{aligned} P &= E\left\{\left[s(t)+s(t+\tau)\right]^2\right\} \\ &= E\left[s^2[t]\right]+E\left[s^2[t+\tau]\right]+2E\left[s(t)s(t+\tau)\right] \\ &= R(0)+R(0)+2R(\tau) \\ &= 2P_0\left[1+r(\tau)\right] \end{aligned} \tag{1}$$

where $r(\tau)$ is the normalized autocorrelation function of $s(t)$.

$$r(\tau)=\frac{R(\tau)}{R(0)} \tag{2}$$

where $R(\tau)$ is the autocorrelation function of $s(t)$. $R(0)$ is the value of autocorrelation function of $s(t)$ at $\tau=0$, which is the average power of $s(t)$.

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$$R(\tau) = E\left[s(t)s(t+\tau)\right]$$

$$R(0) = E\left[s^2(t)\right] \tag{3}$$

The autocorrelation function of Gaussian White Noise has the form of an impulse function. When $\tau \neq 0$ and $r(\tau) = 0$, the average power is written as

$$P = 2P_0\left[1 + r(\tau)\right] = 2P_0 \tag{4}$$

It follows that by using the signal with statistical characteristics of Gaussian white noise to transmit information, $r(\tau)$ is always zero when taking any value of $\tau \neq 0$.

In other words, the communication system would not produce any interference fading. Since there is no interference, the average power of received signal is constant and at this moment, the system is an ideal communication system to resist multipath interference.

The ideal Gaussian white noise bandwidth tends to infinity [11], however, when applied in real, it's impossible to find this ideal Gaussian noise signal. Hence, this can only be the theoretical limit. The multipath resistance of band-limited white noise will be further analyzed below.

Band-limited white noise has the following power spectrum density:

$$S(f) = \begin{cases} \dfrac{P_0}{B} & |f - f_0| \leq \dfrac{B}{2} \\ 0 & |f - f_0| > \dfrac{B}{2} \end{cases} \tag{5}$$

where f_0 and B represent the central frequency and bandwidth of band-limited white noise, respectively, whose unit are all Hz. P_0 is the power of band-limited white noise whose unit is W. The autocorrelation function of band-limited white noise is

$$\begin{aligned} R(\tau) &= \int_{-\infty}^{\infty} S(f) e^{j2\pi f\tau} df = \int_{-f_0-B/2}^{-f_0+B/2} \frac{P_0}{2B} \exp(j2\pi f\tau) df + \int_{f_0-B/2}^{f_0+B/2} \frac{P_0}{2B} \exp(j2\pi f\tau) df \\ &= P_0 \frac{\sin(\pi B\tau)}{\pi B\tau} \cos(2\pi f_0 \tau) \end{aligned} \tag{6}$$

Normalizing the above formula and substituting it into (4), we can obtain

$$P = 2P_0\left[1 + \frac{\sin(\pi B\tau)}{\pi B\tau}\cos(2\pi f_0\tau)\right] \tag{7}$$

The relative fluctuation of received signal power is

$$\frac{P}{2P_0} = 1 + \frac{\sin(\pi B\tau)}{\pi B\tau}\cos(2\pi f_0\tau) \tag{8}$$

For a certain bandwidth B, the minimum value of $P/(2P_0)$ occurs in $\cos(2\pi f_0 t) = -1$. At this time,

$$\left(\frac{P}{2P_0}\right)_{\min} = 1 - \frac{\left|\sin(\pi B\tau)\right|}{\pi B\tau} \tag{9}$$

Because of $\left|\sin(\pi B\tau)\right| \leq 1$, the above equation can be approximated to

$$\left(\frac{P}{2P_0}\right)_{\min} \geq 1 - \frac{1}{\pi B \tau} \tag{10}$$

From upper formula, when the system bandwidth B is large enough, the minimum relative fluctuation $\left[P/(2P_0)\right]_{\min}$ of received signal power can be arbitrarily close to 1, which means multipath fading can be eliminated by increasing system bandwidth.

3. SYSTEM MODELING ANALYSIS IN RAYLEIGH CHANNEL

3.1 Mathematical Model

After reflection and refraction, original single path sending signals compound a cluster of signals, which has $|\tau| < T$ time delay. Its signal obeys Rayleigh distribution and its phase obeys uniform distribution. Rayleigh distribution function is

$$f(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right), x > 0 \tag{11}$$

Its mean and standard deviation is

$$E(x) = \sigma\sqrt{\frac{\pi}{2}} \tag{12}$$

$$D(x) = \sigma^2 \frac{4-\pi}{2} \tag{13}$$

Assuming $m(t)$ is Rayleigh stochastic process whose coefficient is σ, receipt signal can be written as

$$r_i(t) = s_i(t) m(t) \quad (i = 1, 2) \tag{14}$$

Assuming sending signal is $s_1(t)$, the correlation coefficient $\rho_{s_1 r_1}$ between receipt signal $r_1(t) = s_1(t)m(t)$ and local signal $s_1(t)$ is

$$\begin{aligned} \rho_{s_1 r_1} &= \frac{\int_0^T s_1(t) r(t) \mathrm{d}t}{\sqrt{\int_0^T s_1^2(t) \mathrm{d}t} \sqrt{\int_0^{T_s} r^2(t) \mathrm{d}t}} = \frac{\int_0^T s^2{}_1(t) m(t) \mathrm{d}t}{\sqrt{\int_0^T s_1^2(t) \mathrm{d}t} \sqrt{\int_0^T m^2(t) s_1^2(t) \mathrm{d}t}} \\ &= \frac{E[m(t)] \int_0^T s^2{}_1(t) \mathrm{d}t}{\sqrt{E[m^2(t)]} \sqrt{\int_0^T s_1^2(t) \mathrm{d}t} \sqrt{\int_0^T s_1^2(t) \mathrm{d}t}} = \frac{\sigma\sqrt{\frac{\pi}{2}}}{\sqrt{\sigma^2 \frac{4-\pi}{2} + \left(\sigma\sqrt{\frac{\pi}{2}}\right)^2}} = \sqrt{\frac{\pi}{4}} \end{aligned} \tag{15}$$

The correlation coefficient $s_2(t)$ between receipt signal and local signal $\rho_{s_2 r_1}$ is

$$\begin{aligned} \rho_{s_2 r_1} &= \frac{\int_0^T s_2(t) r(t) \mathrm{d}t}{\sqrt{\int_0^T s_2^2(t) dt} \sqrt{\int_0^T r^2(t) \mathrm{d}t}} \\ &= \frac{\int_0^T s_1(t) s_2(t) m(t) \mathrm{d}t}{\sqrt{\int_0^T s_2^2(t) \mathrm{d}t} \sqrt{\int_0^T m^2(t) s_1^2(t) \mathrm{d}t}} \\ &= \frac{E[m(t)] \int_0^T s_1(t) s_2(t) \mathrm{d}t}{\sqrt{E[m^2(t)]} \sqrt{\int_0^T s_1^2(t) dt} \sqrt{\int_0^T s_2^2(t) \mathrm{d}t}} = 0 \end{aligned} \tag{16}$$

In a similar way, $\rho_{s_i r_i} = \sqrt{\pi/4}$, $\rho_{s_1 r_2} = 0$ can be acquired.

After passing Rayleigh channel, the autocorrelation coefficient of sending signal becomes to $\rho_{s_i r_i} = \sqrt{\pi/4}$ from $\rho_{ss} = 1$. It can be approximate that the code energy of sending signal has decreased by $\rho_{s_i r_i}{}^2$. According to the error rate function of Chirp-BOK coherent reception theory, the error rate function of Chirp signal under Rayleigh channel is

$$P_e = \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{\rho_{s_i r_i}{}^2 E_b(1-\rho)}{2n_0}}\right] = \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{\pi E_b}{8n_0}}\right] \quad (17)$$

3.2Simulation Analysis

Figure.1 shows the error rate of Chirp-BOK coherent demodulation under Rayleigh channel when signal bandwidth is $B = 20\text{MHz}$ and symbol period is $T = 6\mu s$. According simulation results, practical curve of bit error rate is consistent with theoretical value, which has around 1dB difference with AWGN. It verifies the validity of foregoing deduction that Chirp-BOK modulation has particular anti-flat fading performance.

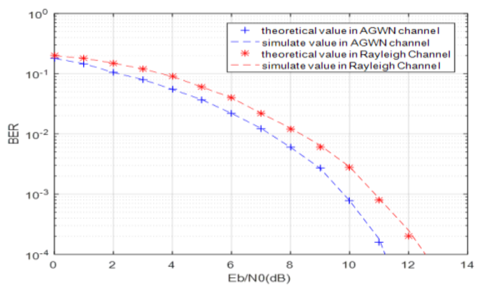

Figure. 1 The error rate of Chirp-BOK modulation under Rayleigh channel

4. SYSTEM MODELING ANALYSIS IN FREQUENCY SELECTIVE FADING CHANNEL

Taking two-path propagation model as an example, this chapter researches the influence from multipath propagation to Chirp-BOK modulation. Regarding main-path signal as sending signal $s_i(t)$ and regarding bypass signal as coherent jamming $j(t)$, receipt signal can be written as

$$r(t) = s_i(t) + j(t) + n(t) \quad (18)(i=1,2)$$

Bringing above formula into the decision function, we can get total bit error rate function of binary signal under coherent jamming:

$$P_e = \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\frac{E_b(1-\rho)}{2n_0}} + \sqrt{\frac{E_j}{n_0}}\rho_{sj}\right) + \operatorname{erfc}\left(\sqrt{\frac{E_b(1-\rho)}{2n_0}} - \sqrt{\frac{E_j}{n_0}}\rho_{sj}\right)\right] \quad (19)$$

ρ is correlation coefficient of encode element which define as:

$$\rho = \frac{\int_0^T s_1(t)s_2(t)\,dt}{\sqrt{\left[\int_0^T s_1^2(t)\,dt\right]\left[\int_0^T s_2^2(t)\,dt\right]}} = \frac{\int_0^T s_1(t)s_2(t)\,dt}{\sqrt{E_1 E_2}} \quad (20)$$

where $E_1 = \int_0^T s_1^2(t)\,dt$ and $E_2 = \int_0^T s_2^2(t)\,dt$ is energy of signal element. ρ_{sj} is element correlation coefficient between jamming and signal which defined as:

$$\rho_{sj} = \frac{\int_0^T j(t) s_d(t) \, \mathrm{d}t}{\sqrt{\left[\int_0^T j(t) \, \mathrm{d}t\right]\left[\int_0^T s_d^2(t) \, \mathrm{d}t\right]}} = \frac{\int_0^T j(t) s_d(t) \, \mathrm{d}t}{\sqrt{E_j \|s_d\|^2}} \tag{21}$$

where $E_j = \int_0^T j^2(t) \, \mathrm{d}t$ is energy of jamming signal element and $s_d(t) = s_1(t) - s_2(t)$, $\|s_d\|^2 = \int_0^T s_d^2(t) \, \mathrm{d}t$ is energy of signal element waveform difference.

The effects of multipath interference on system performance will be discussed in the following two aspects: the change of coherent mediation performance and the attenuation of signal energy when system reception caused by multipath interference.

4.1 The Influence of Coherent Mediation Performance by Multipath

4.1.1 $\mathbf{nT - \frac{1}{B} < \tau < nT + \frac{1}{B}}$

When $\tau = \mathrm{nT}$ (n is positive integer) which means the time delay of bypass signal and main-path signal equals to an integral multiple of element time, receipt signal and jamming signal export pulse overlap passing matching filter. It has maximum jamming effect and bit error rate is higher compared to another situation. Because the spike pulse exported by Chirp signal through matching filter is compressed between $(-1/B, 1/B)$, the pulse partial overlaps when $nT - \frac{1}{B} < \tau < nT + \frac{1}{B}$. Its effect is similar to situation when $\tau = \mathrm{nT}$. Therefore, we only analyze the correlation coefficient when $\tau = \mathrm{nT}$. Element correlation coefficient ρ_{sj} between jamming and signal is

$$\begin{aligned} \rho_{sj} &= \frac{(j, s_d)}{\sqrt{E_j}\|s_d\|} = \frac{\int_0^T \left[a_n g(t-nT) s_2(t) + \bar{a}_n g(t-nT) s_1(t)\right]\left(s_1(t) - s_2(t)\right) \mathrm{d}t}{\left[\int_0^T j^2(t) \, \mathrm{d}t\right]^{1/2} \left[\int_0^T s_d^2(t) \, \mathrm{d}t\right]^{1/2}} \\ &= \frac{\frac{1}{2}(\bar{a}_n - a_n) T}{\sqrt{\frac{1}{2}(\bar{a}_n - a_n) T}\sqrt{T}} = \sqrt{\frac{1}{2}}(\bar{a}_n - a_n) \end{aligned} \tag{22}$$

When $nT - \frac{1}{B} < \tau < nT + \frac{1}{B}$, the bit error rate of Chirp Signal is

$$P_e = \frac{1}{4}\left[\mathrm{erfc}\left(\sqrt{\frac{E_b}{2n_0}} + \sqrt{\frac{E_j}{2n_0}}(\bar{a}_n - a_n)\right) + \mathrm{erfc}\left(\sqrt{\frac{E_b}{2n_0}} - \sqrt{\frac{E_j}{2n_0}}(\bar{a}_n - a_n)\right)\right] \tag{23}$$

$\bar{a}_n$ is the inverse of a_n and $P = \frac{1}{2}$.

The bit error rate curve is shown as Figure.2 when the ratio of main-path energy to bypass energy is 10dB.

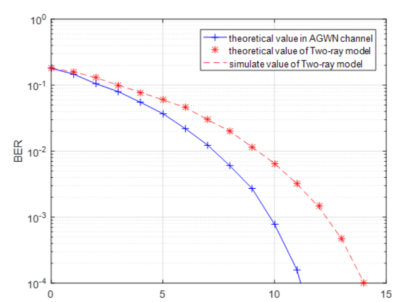

Figure. 2 Two-path channel bit error rate

When there are multiple bypasses satisfied $\mathrm{nT}-\frac{1}{\mathrm{B}}<\tau<\mathrm{nT}+\frac{1}{\mathrm{B}}$, their bit error rate formulas still accord with (23), only plugging energy summation $\mathrm{E_j}$ of multiple interference paths into the equation.

4.1.2 $\mathbf{nT+\frac{1}{B}<\tau<(n+1)T-\frac{1}{B}}$

Not considering the energy attenuation of jamming signal, $\mathrm{nT}+\frac{1}{\mathrm{B}}<\tau<(\mathrm{n}+1)\mathrm{T}-\frac{1}{\mathrm{B}}$ can be approximate to $\frac{1}{\mathrm{B}}<\tau<\mathrm{T}-\frac{1}{\mathrm{B}}$, which means the second path signal is part of two consecutive code elements within the first path signal element time as shown in Figure.3.

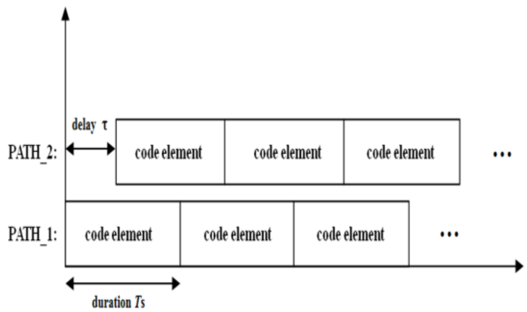

Figure. 3 Two-path channel element overlapping diagram

In the first path signal element time, the second path signal can be expressed as

$$j(t)=\begin{cases}a_{n_1}g_1(t-nT)s_1(t+T-\tau)+\bar{a}_{n_1}g_1(t-nT)s_2(t+T-\tau)\\+a_{n_2}g_2(t-nT+\tau)s_1(t)+\bar{a}_{n_2}g_2(t-nT+\tau)s_2(t)\end{cases} \tag{24}$$

where $a_{n_i}=\begin{cases}0, & P\\+1, & 1-P\end{cases}$ and $\bar{a}_{n_i}$ is radix-minus-one complement of a_n. $\bar{a}_{n_i}=\begin{cases}+1, & P\\0, & 1-P\end{cases}$ and $P=\dfrac{1}{2}$. $g_1(t)$ and $g_2(t)$ are rectangular pulses having τ and $T-\tau$ duration respectively.

When time delay is τ, frequency shift is $\mu\tau$. Then

$$j(t)=j_1(t)+j_2(t) \tag{25}$$

When $\tau<\mathrm{T}$, $\mathrm{j_1(t)}$ in upper formula is prior element (τ,T) part of main path and $\mathrm{j_2(t)}$ is current element $(0,\tau)$ part of main path. When $\mathrm{nT}<\tau<(\mathrm{n}+1)\mathrm{T}$, setting main-path current element as the L_{th} element, $\mathrm{j_1(t)}$ is $\mathrm{L}-\mathrm{n}-1_{th}$ element (τ,T) part and $\mathrm{j_2(t)}$ is $\mathrm{L}-\mathrm{n}_{th}$ element part. After time domain transformation, definition domain (τ,T) of $\mathrm{j_2(t)}$ changes to $(0,\mathrm{T}-\tau)$.

$$j_1(t)=\begin{cases}a_{n_1}\cos\left(2\pi(f_0+\mu\tau)t+\pi\mu t^2\right)\\+\bar{a}_{n_1}\cos\left(2\pi(f_0+B-\mu\tau)t-\pi\mu t^2\right)\end{cases},0<t<\tau \tag{26}$$

$$j_2(t)=\begin{cases}a_{n_2}\cos\left(2\pi f_0t+\pi\mu t^2\right)\\+\bar{a}_{n_2}\cos\left(2\pi(f_0+B)t-\pi\mu t^2\right)\end{cases},0<t<T-\tau \tag{27}$$

Corresponding to $\mathrm{j_1(t)}$ and $\mathrm{j_2(t)}$, $\mathrm{s_d(t)}$ is split into $\mathrm{s_{d_1}(t)}$ and $\mathrm{s_{d_2}(t)}$.

$$s_{\mathrm{d}}(t)=s_1(t)-s_2(t)=s_{\mathrm{d}_1}(t)+s_{\mathrm{d}_2}(t) \tag{28}$$

$$s_{d_1}(t)=\begin{cases}\cos\left(2\pi f_0 t+\pi\mu t^2\right)\\ -\cos\left(2\pi\left(f_0+B\right)t-\pi\mu t^2\right)\end{cases},0<t<\tau \tag{29}$$

$$s_{d_2}(t)=\begin{cases}\cos\left(2\pi\left(f_0+\mu\tau\right)t+\pi\mu t^2\right)\\ -\cos\left(2\pi\left(f_0+B-\mu\tau\right)t-\pi\mu t^2\right)\end{cases},0<t<T-\tau \tag{30}$$

The time domain of $j_1(t)$ is different from $s_{d_2}(t)$ and same as the $j_2(t)$ and $s_{d_1}(t)$ which can be regarded as orthogonality. Then the element correlation coefficient ρ_{sj} of jamming and signal is

$$\begin{aligned}\rho_{sj}&=\frac{(j,s_d)}{\sqrt{E_j}\,\|s_d\|}=\frac{\int_0^T j(t)s_d(t)\mathrm{d}t}{\left[\int_0^T j^2(t)\mathrm{d}t\right]^{1/2}\left[\int_0^T s_d^2(t)\mathrm{d}t\right]^{1/2}}\\ &=\frac{\int_0^\tau j_1(t)s_{d_1}(t)\mathrm{d}t+\int_0^{T-\tau} j_2(t)s_{d_2}(t)\mathrm{d}t}{\left[\int_0^T j^2(t)\mathrm{d}t\right]^{1/2}\left[\int_0^T s_d^2(t)\mathrm{d}t\right]^{1/2}}\end{aligned} \tag{31}$$

$\left|\int_0^\tau j_1(t)s_{d_1}(t)\mathrm{d}t\right|$ and $\left|\int_0^{T-\tau} j_2(t)s_{d_2}(t)\mathrm{d}t\right|$ can be transferred to

$$\left|\begin{aligned}&\int_0^{T_i}\cos\left(2\pi f_i t+\pi\mu t^2\right)\cos\left(2\pi\left(f_i+B_i\right)t\pm\pi\mu t^2\right)\mathrm{d}t\\ &-\int_0^{T_i}\cos\left(2\pi f_i t-\pi\mu t^2\right)\cos\left(2\pi\left(f_i+B_i\right)t\pm\pi\mu t^2\right)\mathrm{d}t\end{aligned}\right| \tag{32}$$

where $i=0,1, B_1=\mu\tau$, $T_1=\tau$ and $T_2=T-\tau$. f_1 is center frequency and $f_1\ll B$. Therefore, the value of f_1 varies within a certain range and does not affect correlation coefficient ρ_{sj}. The upper formula can be represented as

$$\left|\varphi_{1i}(t)-\varphi_{2i}(t)\right| \tag{33}$$

When having same bandwidth, element length and product of time width and bandwidth, $\varphi_{1i}(t)$ is integral of common bandwidth Chirp signals having μ and $-\mu$ frequency modulation slope respectively. In other words, it is to calculate the cross-correlation of Up-Chirp and Down-Chirp.

$$\varphi_{1i}(t)=\int_0^{T_i}\cos\left(2\pi f_i t+\pi\mu t^2\right)\cos\left(2\pi\left(f_i+B_i\right)t-\pi\mu t^2\right)\mathrm{d}t \tag{34}$$

In (33), $\varphi_{2i}(t)$ is integral of Chirp signals whose center frequency differs B_i when bandwidth, element length, product of time width and bandwidth and frequency modulation slope are same. Considering the insensitivity to frequency offset of Chirp signal, $\varphi_{2i}(t)$ can be ignored relative to $\varphi_{1i}(t)$.

$$\varphi_{2i}(t)=\int_0^{T_i}\cos\left(2\pi f_i t\pm\pi\mu t^2\right)\cos\left(2\pi\left(f_i+B_i\right)t\pm\pi\mu t^2\right)\mathrm{d}t \tag{35}$$

Uniting cross-correlation coefficient of Up-Chirp and Down-Chirp,

$$\left|\rho_{s_1s_2}=\frac{\int_0^T s_1(t)s_2(t)\mathrm{d}t}{\int_0^T s_i^2(t)\mathrm{d}t}\right|\le 0.0657 \tag{36}$$

(33) can be simplified to

$$
\begin{aligned}
\rho_{sj} &= \frac{\left(\bar{a}_{n_1} - a_{n_1}\right)\varphi_{11}(t) + \left(\bar{a}_{n_2} - a_{n_2}\right)\varphi_{12}(t)}{\left[\int_0^T j^2(t)\,\mathrm{d}t\right]^{1/2}\left[\int_0^T s_{\mathrm{d}}^2(t)\,\mathrm{d}t\right]^{1/2}} \\
&= \frac{\left(\bar{a}_{n_1} - a_{n_1}\right)\varphi_{11}(t) + \left(\bar{a}_{n_2} - a_{n_2}\right)\varphi_{12}(t)}{\left[\int_0^T s_i^2(t)\,\mathrm{d}t\right]^{1/2}\left[\int_0^T \left(s_1^2(t) + s_2^2(t)\right)\mathrm{d}t\right]^{1/2}} \\
&= \frac{1}{\sqrt{2}}\left[\left(\bar{a}_{n_1} - a_{n_1}\right)\frac{T_1}{T}\rho_{s_1 s_2} + \left(\bar{a}_{n_2} - a_{n_2}\right)\frac{T_2}{T}\rho_{s_1 s_2}\right] \\
&= \pm\frac{3\rho_{s_1 s_2}}{4\sqrt{2}} = \pm 0.0348
\end{aligned} \tag{37}
$$

Therefore, when $nT + \frac{1}{B} < \tau < (n+\mathbf{1})T - \frac{1}{B}$, bit error rate of Chirp signal is

$$
P_{\mathrm{e}} = \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\frac{E_{\mathrm{b}}}{2n_0}} + 0.0348\sqrt{\frac{E_j}{n_0}}\right) + \operatorname{erfc}\left(\sqrt{\frac{E_{\mathrm{b}}}{2n_0}} - 0.0348\sqrt{\frac{E_j}{n_0}}\right)\right] \tag{38}
$$

When there are multiple bypasses satisfying $nT + \frac{1}{B} < \tau < (n+\mathbf{1})T - \frac{1}{B}$, we only need to substitute energy summation E_j of multiple disturbing paths into (38). In this paper, main path is set to the path having smallest time delay by default and remained unchanged. Figure.4 demonstrates the bit error rate curves of energy summation ratio of main path and bypass are -4.8dB and -10.8dB.

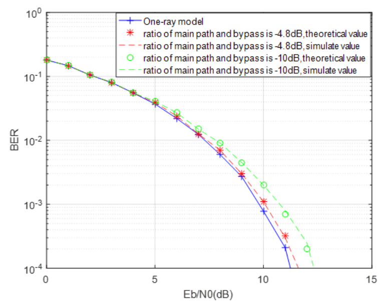

Figure. 4 Bit error rate of bypass without noises

In above deduction and simulation, regarding bypass signal as jamming, the added SNR is the main-path SNR not the overall signal SNR. When energy summation ratio of main path and bypass is -4.8dB, bypass signal's disturbance can be ignored and bit error rate curve can be approximated to bit error rate function of Chirp-Bok coherent reception theory:

$$
P_{\mathrm{e}} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_{\mathrm{b}}}{2n_0}(1-\rho)}\right) \tag{39}
$$

4.2 The Influence of Signal Energy by Multipath

This part analyzes signal energy attenuation caused by multipath affects system performance. Setting the ratio of main path energy and bypass energy -4.8dB as boundary, main path and bypass are treated as signal as a whole. According to the analysis in previous section, bypass jamming can be ignored this moment. The effect of system bit error rate is only connected with attenuation of main-path SNR relative to total SNR. As shown in Figure.5, when the ratio of main path energy and bypass energy is -4.8dB, the ratio of main path SNR and total SNR is -6.02dB. When the ratio of main path energy and bypass energy is 0dB, main path SNR is lower -3dB than total SNR. While the ratio of main path energy and bypass energy is 3dB, the SNR ration of main path and total is -1.76dB.

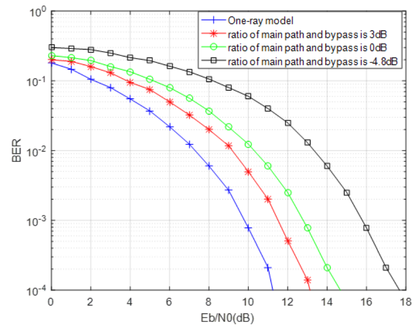

Figure. 5 Bit error rate of bypass with noises

In conclusion, when time delays of multiple paths meet $nT - \frac{1}{B} < \tau < nT + \frac{1}{B}$ and the ratio of main path energy and the sum energy of all bypasses is 10dB, the performance reduces 2dB at 10^{-4} bit error rate. In this case, system performance is greatly affected. Only when the appearance probabilities of path having all kinds of time delay are equal, the path, satisfying $nT - \frac{1}{B} < \tau < nT + \frac{1}{B}$ and having just 1.5% appearance probability on the basis of $^{2/B}/_{T} = 1.5\%$, can be ignored.

While time delays of multiple paths all satisfy $nT + \frac{1}{B} < \tau < (n + \mathbf{1})T - \frac{1}{B}$, the ratio of main path energy and the sum energy of all bypasses is 0dB. At 10^{-4} bit error rate level, the performance decreases 3dB which means other receiving technologies may not be used. When the ratio of main path energy and the sum energy of all bypasses is -4.8dB, the performance reduces 6dB and it is necessary to consider the utilization of bypass energy such as diversity techniques.

5. SUMMARY

In this paper, the performance of Chirp-BOK modulation is analyzed in multipath channel. The performance analysis is not only demonstrated by MATLAB simulation but also supported by the mathematical model deduction. Finally, it is concluded that the use of Chirp-BOK modulation can effectively suppress multipath interference.

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