

References

- [1] Andrews, L. C.: *Elementary Partial Differential Equations with Boundary Value Problems* (Academic Press, Orlando, 1986).
- [2] Andrews, L. C. and R. L. Phillips: *Laser Beam Propagation through Random Media* (SPIE Optical Engineering Press, Bellingham, 1998).
- [3] Arfken, G.: *Mathematical Methods for Physicists*, 3rd ed. (Academic Press, San Diego, 1985).
- [4] Beckmann, P.: *Probability in Communication Engineering* (Harcourt Brace & World, New York, 1967).
- [5] Bell, W. W.: *Special Functions for Scientists and Engineers* (Van Nostrand, London, 1968).
- [6] Boggess, A. and F. J. Narcowich, *A First Course in Wavelets with Fourier Analysis* (Prentice-Hall, Upper Saddle River, 2001).
- [7] Born, M. and E. Wolf: *Principles of Optics*, 6th ed. (Cambridge University Press, Cambridge, 1980).
- [8] Boyce, W. and R. DiPrima: *Elementary Differential Equations*, 5th ed. (Wiley, New York, 1992).
- [9] Cooley, J. W. and J. W. Tukey, "An algorithm for the machine calculation of complex Fourier series," *Math. Comp.* **19**, 297-301 (1965).
- [10] Davenport, Jr., W. B. and W. L. Root: *An Introduction to the Theory of Random Signals and Noise* (IEEE Press, New York, 1987).
- [11] Debnath, L. "Generalized calculus and its applications," *Int. J. Math. Educ. Sci. Technol.* **9**, 399-416 (1978).
- [12] Debnath, L. and P. Mikusinski: *Introduction to Hilbert Spaces with Applications* (Academic Press, San Diego, 1990).
- [13] Erdélyi, A., W. Magnus, F. Oberhettinger, and F. G. Tricomi: *Tables of Integral Transforms* (in two volumes) (McGraw-Hill, New York, 1954).
- [14] Gagliardi, R. M. and S. Karp.: *Optical Communications*, 2nd ed. (John Wiley & Sons, New York, 1995).
- [15] Gaskill, J. D.: *Linear Systems, Fourier Transforms, and Optics* (Wiley & Sons, New York, 1978).
- [16] Goodman, J. W.: *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968).
- [17] Ince, E. I.: *Ordinary Differential Equations*. (Dover, New York, 1956).
- [18] Jury, E. I.: *Theory and Application of the Z-Transform* (Wiley & Sons, New York, 1964).
- [19] Kingston, R. H.: *Optical Sources, Detectors, and Systems: Fundamentals and Applications* (Academic Press, San Diego, 1995).
- [20] Lohmann, A. W., D. Mendlovic, and Z. Zalevsky.: "Fractional transformations in optics," in *Progress in Optics XXXVIII*, E. Wolf, ed., Elsevier Science B. V. (1998).
- [21] Mendlovic, D. and H. M. Ozaktas: "Fractional Fourier transforms and their optical implementation: I," *J. Opt. Soc. Am. A* **10**, 1875-1881 (1993); "Fractional Fourier transforms and their optical implementation: II," *J. Opt. Soc. Am. A* **10**, 2522-2531 (1993).

- [22] Middleton, D.: *An Introduction to Statistical Communication Theory* (McGraw-Hill, New York, 1960).
- [23] Noll, R. J.: "Zernike polynomials and atmospheric turbulence," *J. Opt. Soc. Am.* **66**, 207-211 (1976).
- [24] Papoulis, A.: *Probability, Random Variables, and Stochastic Processes* (McGraw-Hill, 1965).
- [25] Resnikoff, H. L. and R. O. Wells, Jr.: *Wavelet Analysis: The Scalable Structure of Information* (Springer-Verlag, New York, 1998).
- [26] Rice, S. O.: "The mathematical analysis of random noise," *Bell Sys. Tech. J.* **23**, 282-332 (1944); "Statistical properties of a sine wave plus random noise," *Bell Sys. Tech. J.* **27**, 109-158 (1948).
- [27] Roggemann, M. C. and B. Welsh: *Imaging Through Turbulence* (CRC Press, Boca Raton, 1996).
- [28] Sasiela, R. J.: *Electromagnetic Wave Propagation in Turbulence* (Springer, New York, 1994).
- [29] Siegman, A. E.: *Lasers* (University Science, Mill Valley, 1986).
- [30] Sokolnikoff, I. S.: *Mathematical Theory of Elasticity*, 2nd ed. (McGraw-Hill, New York, 1956).
- [31] Springer, M. D. and W. E. Thompson, "The distribution of products of beta, gamma, and Gaussian random variables," *SIAM J. Appl. Math.* **18**, 721-737 (1970).
- [32] Stavroudis, O. N.: *The Optics of Rays, Wavefronts, and Caustics* (Academic Press, New York, 1972).
- [33] Tyson, R. K.: *Principles of Adaptive Optics* (Academic Press, San Diego, 1991).
- [34] Watson, G. N.: *A Treatise on the Theory of Bessel Functions*, 2nd ed. (Cambridge University Press, London, 1952).
- [35] Weinstock, R.: *Calculus of Variations with Applications to Physics and Engineering* (Dover, New York, 1974).
- [36] Wheelon, A. D.: *Electromagnetic Scintillation: I Geometrical Optics* (Cambridge University Press, Cambridge, 2001).
- [37] Ziemer, R. E. and W. H. Tranter: *Principles of Communications: Systems Modulation, and Noise*, 5th ed. (Wiley & Sons, New York, 2002).
- [38] *Applications of Walsh Functions, 1970 Proceedings, Symposium and Workshop*, sponsored by Naval Res. Lab. and U. Maryland, Doc. AD 707431, Nat. Tech. Infor. Ser., Virginia.

Index

A

- ABCD* matrices, 767
- Absolute derivative, 258
- Absolutely integrable, 455
- Acceleration, 166
 - centripetal, 167
 - coriolis, 167, 170
 - gravitational, 14
- Addition of
 - complex numbers, 274
 - matrices, 111
 - random variables, 678, 688
 - vectors, 147
- Admissible function, 547
- Airy
 - equation, 56, 93
 - functions, 94
- Ampère's law, 208
- Amplitude spectrum, 462
- Analytic
 - function, 291
 - signal representation, 475
- Angular frequency, 416, 462
- Annular region, 327
- Annulus, 347
- Arc length, 163, 182
- Argument, 277
 - principal value, 277
- Associated Laguerre polynomial, 80
- Asymptotic stability, 130
- Autocorrelation function. *See*
 - correlation function
- Autocovariance function. *See*
 - covariance function
- Auxiliary condition, 6
 - See also* Boundary conditions and Initial conditions
- Auxiliary equation, 20, 123
- Average,
 - ensemble, 700
 - power, 405, 715
 - time, 703
 - (*See also*, mean value)
- Axioms of probability, 639

B

- Base vectors,
 - Cartesian, 148, 226
 - curvilinear, 248
 - reciprocal, 227
 - rectilinear, 226
- Basis, 134
- Bayes's theorem, 669
- Beam. *See* Elastic beam
- Bernoulli,
 - Daniel, 62, 590
 - James, 2, 546, 590
 - John, 2
- Bernoulli equation, 387
- Bessel equation, 50, 84
 - related to, 56
- Bessel, Friedrich W., 62
- Bessel functions,
 - Airy, 94
 - first-kind, 52, 84, 617
 - Hankel, 93
 - modified, 88
 - spherical, 92
 - second-kind, 55, 86
- Bessel series, 438
- Beta function, 70
- Bilinear formula, 446
- Bilinear transformation, 311
- Binomial
 - distribution, 647
 - series, 101
- Binormal vector, 164
- Bit error rate, 756
- Bivariate distribution, 665
- Blasius, theorem of, 395
- Borel, Félix E. E., 638
- Boundary conditions, 420
 - forced, 547
 - implicit, 609
 - natural, 557, 561
 - periodic, 431, 609
- Boundary value problem, 442
 - eigenvalue problem, 420
- Brachistochrone problem, 553

- Branch
 - cut, 302
 - point, 302
- Buckling problem, 742
 - Euler load, 743
- C**
- Canonical form, 603
- Carson's rule, 747
- Carrier-to-noise ratio, 752
 - (*See also*, signal-to-noise ratio)
- Cartesian
 - coordinates, 146
 - tensor notation, 156, 216
- Catenary, 553
- Cauchy, Augustin-Louis, 272
- Cauchy
 - distribution, 655
 - inequality, 337
 - integral formula, 332, 336
 - integral theorem, 325
 - principal value, 365
- Cauchy-Euler equation, 31
- Cauchy-Riemann equations, 198, 289
 - polar form, 291
- Causal function, 473
- Central limit theorem, 691
- Central moments, 650
- Centripetal acceleration, 167
- Characteristic function, 653
 - normal distribution, 653
- Characteristic
 - value. *See* Eigenvalue
- Chebyshev
 - equation, 82
 - polynomials, 81
- Chi-square distribution, 665
- Christoffel symbols, 251
- Circle of convergence, 342
- Circulation, 182, 394
- Cis, 278
- Classification
 - of ODEs, 3
 - of PDEs, 591
- Clifford, W. L., 144
- Coefficients of a DE, 17
- Column,
 - buckling, 742
 - vector, 121
- Comb function, 65
- Compatibility condition, 199
- Complementary error function, 72
- Complex
 - conjugate, 275
 - function, 281
 - line integral, 321
 - number, 273
 - plane, 275
 - potential, 388
 - series, 339
 - velocity, 387
- Component
 - contravariant, 222
 - covariant, 222
 - vector, 146
- Conditional
 - density functions, 656, 668
 - expected value, 657
 - probability, 655, 668
- Conductivity constant, 599
- Confluent hypergeometric
 - equation, 97
 - functions, 97
- Conformal mapping, 313
- Conjugate complex number, 275
- Connected set, 183, 282
 - simply, 183
- Conservation of
 - energy principle, 554
 - mass, 175
 - thermal energy, 599
- Conservative field, 183
- Constraints, 565
- Contiguous functions, 96
- Continuous
 - distribution, 641
 - random variable, 640
- Contour, 321
 - closed, 322
 - indented, 371
 - simple, 321
- Contour integral, 321
- Contraction, 224
- Contravariant tensor, 222

- Convective heat transfer, 611
 - Convergence, 339
 - circle (radius) of, 342
 - pointwise, 340, 409
 - uniform, 340
 - Convolution theorem,
 - discrete Fourier, 532
 - Fourier transform, 470
 - Laplace transform, 505
 - Mellin, 522
 - Coordinates,
 - Cartesian, 146
 - curvilinear, 245
 - cylindrical, 246
 - polar, 608
 - rectilinear, 218
 - spherical, 268
 - Coriolis acceleration, 167
 - Correlation, 673
 - coefficient, 674
 - function, 701
 - Cosine,
 - integral representation, 457
 - inverse, 304
 - series, 412
 - transform, 460
 - Covariance function, 701
 - normalized, 701
 - table of, 714
 - Covariance, 673
 - matrix, 723
 - Covariant
 - derivative, 255
 - tensor, 222
 - Critical
 - damping, 738
 - point, 129, 314
 - Cross
 - correlation function, 708
 - correlator, 710
 - covariance function, 708
 - spectral density, 721
 - Cross product, 150, 235
 - Cumulative distribution function, 640
 - conditional, 655
 - Curl, 176
 - Curvature, 164
 - Curve
 - arc length of, 163, 182
 - rectifiable, 324
 - simple, 321
 - Curvilinear coordinates, 245
 - Cycloid, 555
 - Cylinder, flow around, 396
 - Cylindrical coordinates, 246
- D**
- d'Alembert, Jean Le Rond, 590
 - d'Alembert solution, 602
 - Damped motion, 735
 - dc component, 419, 715
 - Deformation of path, 326
 - Degrees of freedom, 573
 - Deleted neighborhood, 282
 - Delta,
 - function. *See* Impulse function
 - Kronecker, 157, 217
 - DeMoivre formula, 279
 - Density function, 642
 - table of, 647
 - Dependent, linearly, 19, 135
 - Derivative of a
 - complex function, 287
 - vector function, 161
 - Detection,
 - probability of, 756
 - threshold, 754
 - Determinant, 112
 - Dido, problem of, 566
 - Difference equations, 536
 - Differential equation, 3
 - Airy, 93
 - Bessel, 50, 84
 - Cauchy-Euler, 31
 - Cauchy-Riemann, 289
 - confluent hypergeometric, 97
 - constant coefficients, 20
 - elliptic, 592
 - Helmholtz, 618, 762
 - Hermite, 79
 - homogeneous, 9, 17
 - hyperbolic, 592
 - hypergeometric, 96
 - Laguerre, 80

- Laplace, 177, 604
- Legendre, 40, 76
- linear, 4, 8
- nonhomogeneous, 9, 24, 28
- nonlinear, 4
- ordinary, 4
- parabolic, 592
- partial, 591
- separable, 7, 594
- Sturm-Liouville, 425, 431, 432
- Differential operator, 177
- Diffraction pattern,
 - circular slit, 517
 - square aperture, 75
- Diffusion equation. *See* Heat equation
- Diffusivity constant, 593
- Digamma (ψ) function, 71
- Dirac delta function. *See* Impulse function
- Dirac, Paul A. M., 65
- Direction angles, 147
- Directional derivative, 171
- Dirichlet
 - condition, 584, 593
 - problem, 198
- Discrete
 - Fourier transform, 529
 - random variable, 640, 644
 - Walsh transform, 540
 - Z-transform, 533
- Distribution,
 - conditional, 656, 668
 - continuous, 641
 - discrete, 644
 - joint, 665, 698
 - marginal, 666
 - table of, 647
- Divergence, 173
 - theorem, 202
- Domain, 183, 282
- Dot product, 148, 235
- Doublet, 392
- Drag, 396

- E**
- Eigenfunction, 420
 - expansion, 428, 447
 - orthogonality of, 427
- Eigenvalue, 116, 241, 420
 - problem, 116, 124, 420, 570
- Eigenvector, 117, 124, 241
- Eikonal equation, 758
- Einstein, A., 144
- Einstein summation rule, 156
- Elastic
 - beam, 580
 - membrane, 583
 - string, 580, 600
- Electromechanical analogies, 741
- Elliptic equation. *See* Laplace's equation
- Elliptic integral, complete, 494
- Energy, 405
 - signal, 406, 461
 - spectral density, 463
- Ensemble, 698
 - average, 700
- Entire function, 295
- Envelope, 737, 748
- Equidimensional equation. *See* Cauchy-Euler equation
- Equilibrium point. *See* Critical point
- Equipotential
 - curves, 306, 383
 - surfaces, 170
- Ergodic process, 704
- Error function, 12, 72
 - complementary, 72
- Essential singularity, 353
- Euclidean space, 136
- Euler
 - buckling load, 743
 - constant, 71
 - formulas, 278
- Euler-Lagrange equation, 548, 561, 563
 - system of, 559
- Euler, Leonhard, 62, 404, 546, 590
- Even
 - function, 411
 - periodic extension, 414
- Event, 639
- Existence-uniqueness theorem, 6, 18
- Expansion theorem, 137, 428
- Expected value, 649

- conditional, 657
- Exponential
 - function, complex, 295
 - order, 497
- Extension, periodic, 413
- Extremal, 547
- F**
- Fade, 728
- False alarm, 754
 - probability of, 756
- Faraday's law, 207
- Fermat, Pierre de, 555, 638
- Fermat's principle, 555
- Fibonacci sequence, 544
- Field,
 - conservative, 183
 - irrotational, 387
 - scalar, 170
 - vector, 173
- Fluid
 - flow, 175, 386
 - ideal, 387
 - incompressible, 387
- Flux, 189, 599
- Forced oscillations, 734
- Forcing function, 8
 - See also* Input function
- Fourier
 - Bessel series, 438
 - coefficients, 408
 - cosine integral, 457
 - cosine series, 412
 - integral representation, 456
 - integral theorem, 455
 - Legendre series, 434
 - law, 599
 - series, 405
 - series, complex, 416
 - series, generalized, 428, 611, 774
 - sine integral, 457
 - sine series, 412
- Fourier, Joseph, 404, 590
- Fourier transform, 458, 461, 622
 - cosine, 460
 - discrete, 529
 - fractional, 483
 - inverse, 458, 461
 - properties of, 467
 - sine, 460
 - table of, 464
 - two-dimensional, 477
- Fractional-order
 - derivatives, 483
 - Fourier transform, 483
- Free oscillations, 735
- Frenet-Serret formulas, 164, 759
- Frequency,
 - angular, 416, 462
 - fundamental, 419
 - modulation, 745
 - spectrum, 419
- Fresnel integrals, 73
- Frobenius, Georg, 44
- Frobenius method, 44
- Function,
 - analytic, 291
 - complex, 281
 - multivalued, 300
 - vector, 173
- Functional, 547
 - quadratic, 567
- Fundamental
 - frequency, 419, 601
 - matrix, 122
 - solutions, 18
 - theorem of calculus, 330
- G**
- Galilei, Galileo, 144
- Gamma distribution, 69, 647, 649
 - moments of, 69
- Gamma function, 50, 67
 - incomplete, 68
- Gauss, Carl F., 62, 95, 144
- Gauss divergence theorem, 202
- Gauss's law, 208
- Gaussian
 - distribution. *See* Normal distribution
 - function, 65
- General solution,
 - first-order, 9
 - second-order, 19, 24

- partial DE, 591
- system of linear DEs, 124, 132
- Generalized
 - coordinates, 573
 - force components, 263
 - Fourier series, 428, 611, 774
 - hypergeometric functions, 98
- Generating function, 77, 84
- Geometric series, 340
- Geometrical optics, 756
 - eikonal equation, 758
 - ray equation, 757
- Gibbs, J. Williard, 144
- Gibbs' phenomenon, 410
- Gradient, 171
- Green
 - formulas, 199
 - theorem, 194, 325
- Green, George, 144, 404, 590
- Green's function, 12, 438, 444
 - bilinear formula for, 446
 - one-sided, 439
- H**
- Half-plane,
 - Dirichlet problem for, 379
 - Neumann problem for, 379
- Half-range expansions, 413
- Half-wave rectifier, 662
- Hamilton, William Rowan, 144, 274
- Hamilton principle, 572
- Hankel functions, 93, 619
- Hankel transform, 513, 630, 762
 - inverse, 514
 - properties of, 516
 - table of, 517
- Harmonic
 - conjugate, 294
 - function, 197, 293
 - motion, 601
 - oscillator, 577
- Heat
 - conductivity constant, 599
 - equation, 592
 - flux, 599
- Heaviside expansion formula, 508
- Heaviside, Oliver, 144
- Heaviside unit function. *See* step function
- Helmholtz equation, 618, 762
- Hermite, Charles, 62
- Hermite
 - equation, 79
 - polynomials, 78
- Hermite-Gaussian function, 484, 765
- Hermitian, 462
- Hilbert space, 138
- Hilbert transform, 473
 - table of, 477
- Homogeneous
 - DE, 9, 17, 18
 - solution, 9
 - systems of DEs, 122, 123
- Huygens, Christian, 2
- Huygens-Fresnel integral, 764
 - generalized, 480, 769
- Hyperbolic
 - equation. *See* Wave equation
 - functions, 299
- Hypergeometric
 - equation, 96
 - functions, 95
- I**
- Ideal fluid, 387
- Identity
 - matrix, 113
 - transformation, 220
- Image
 - point, 283
 - plane, 283
- Imaginary
 - axis, 275
 - part of, 274
- Implicit boundary condition, 609
- Improper integral, 365
- Impulse function, 65
 - sifting property of, 66
- Impulse response function, 12, 472, 511
- Incomplete gamma function, 68
- Incompressible flow, 175
- Inconsistent equations, 115
- Indented contour, 371
- Independence of path, 183, 328

- Independent
 - events, 656
 - random variables, 672
 - solutions, 19
- Indicial equation, 45
- Inequality
 - Cauchy, 337
 - Schwarz, 137
 - triangle, 137
- Infinite divisibility, 690
- Infinity in complex plane, 310
- Initial
 - condition, 6, 11
 - value problem, 6, 439, 508, 734
- Inner product, 136, 139
 - space, 135
- Input function, 8, 17
- Integral,
 - contour, 321
 - Fourier, 456
 - improper, 365
 - line, 179
 - surface, 185
 - transform, 496
 - volume, 200
- Interior point, 282
- Intrinsic derivative, 259
- Invariant, 222
- Inverse
 - hyperbolic functions, 304
 - of a matrix, 114
 - trigonometric functions, 304
- Irregular singular point, 44
- Irrrotational fluid, 387
- Isolated singularity, 353
- Isoperimetric problem, 564
- Isotherms, 383

- J**
- Jacobian, 191, 201, 219, 245, 314, 687
- Joint
 - distribution, 665, 698
 - moments, 673
- Jointly stationary, 709

- K**
- K* distribution, 684
- Kepler, Johannes, 168
- Kepler's laws, 169
- Kernel, 496
- Kirchoff's laws, 741
- Kolmogorov, Andrei N., 638
- Kramers-Krönig relations, 476
- Kronecker delta, 157, 217
- Kummer, Ernst E., 62
- Kummer functions. *See* Confluent hypergeometric functions
- Kutta-Joutkowski lift formula, 399

- L**
- Lagrange
 - equation, 80
 - multiplier, 566
- Lagrange, Joseph Louis, 62, 590
- Lagrangian, 572
- Laguerre
 - associated polynomials, 80
 - equation, 80
 - polynomials, 80
- Laplace equation, 177, 604
 - Dirichlet problem, 605
 - Neumann problem, 605
- Laplace, Pierre Simon de, 590
- Laplace transform, 496, 624
 - complex inversion formula, 507
 - inverse, 497, 503
 - properties of, 502, 504
 - table of, 501
- Laplacian, 177, 593
- Laurent
 - series, 346
 - theorem, 349
- Lebesgue, Henri, 638
- Legendre, Adrien M., 62, 590
- Legendre
 - equation, 40, 76
 - functions of the second kind, 42
 - polynomials, 41, 76
 - series, 434
- Leibniz, Gottfried Wilhelm von, 2, 546
- Length
 - of a curve, 163
 - of a vector, 145
- Level curve, 306, 383

- Levi-Civita, T., 144
 L'Hôpital, G. F. A., 546
 L'Hôpital's rule, 55
 Lift, 396
 Limit,
 complex function, 284
 distribution, 690
 Linear
 dependence, 19, 135
 differential equation, 8, 17
 independence, 19, 135
 mapping, 307
 shift-invariant system, 472, 479
 system, 114, 471, 718,
 vector space, 133
 Line integral,
 complex, 321
 vector, 179
 Liouville, Joseph, 404
 Liouville theorem, 338
 Logarithm function, complex, 301
 principal branch, 301
- M**
- Maclaurin series, 344
 Magnitude of a
 complex number, 276
 vector, 145
 Mapping, 283
 bilinear, 311
 conformal, 313
 fixed point, 312
 linear, 307
 one-to-one, 296
 reciprocal, 308
 Marcum Q -function, 682
 Marginal
 density, 666, 667
 distribution, 666
 Markov, Andrei A., 638
 Matrix, 110
 eigenvalue problem, 116
 inverse, 114
 nonsingular, 114
 properties, 111
 singular, 114
 skew-symmetric, 119
 symmetric, 113, 119
 transpose, 113
 Maximum likelihood,
 estimate, 671
 principle of, 671
 Maximum modulus theorem, 338, 611
 Maxwell, James C., 144
 Maxwell's equations, 206
 Mean value, 650, 700
 Mellin transform, 519
 inverse, 519
 properties of, 521
 table of, 522
 Metric
 space, 135
 tensor, 228, 248
 Mixed
 random variables, 644
 triple product, 151, 235
 ML inequality, 323
 Möbius transformation. *See* Bilinear transformation
 Modified Bessel functions, 88
 Modulus of complex number, 276
 Moments,
 central, 650
 of a distribution, 650
 generating function for, 654
 joint, 673
 Morera's theorem, 337
 Multiplication rule, probability, 655
 Multivalued function, 300
 Multivariate distribution, 723
- N**
- Narrowband noise, 747, 752
 Necessary condition, 546, 549, 556
 Negative exponential distribution, 643
 Neighborhood, 282
 Neumann
 condition, 593
 problem, 198
 Newton, Sir Isaac, 2, 546
 Newton second law of motion, 2
 Noise,
 narrowband, 747, 752
 white, 720

- Nonhomogeneous
 - DE, 9, 24, 28
 - solution. *See* Particular solution
 - systems of DEs, 131
 - Nonlinear DE, 4
 - Nonsingular matrix, 114
 - Nontrivial solution, 420
 - Norm, 136, 139, 145, 427
 - Normal distribution, 73, 646
 - bivariate, 665
 - moments of, 651, 654
 - multivariate, 723
 - Normal form, 8, 17
 - Normalized
 - covariance function, 701
 - vector, 145
 - Normal random variable, 646, 647
 - characteristic function for, 653
 - density function for, 73, 646
 - moments of, 651, 654
 - Normal vector, 164, 186
- O**
- Odd
 - function, 411
 - periodic extension, 414
 - One-sided Green's function, 439
 - One-to-one mapping, 296
 - Open set, 183, 282
 - Order of DE, 4
 - Ordinary
 - DE, 4
 - point, 36
 - Oriented surface, 204
 - Orthogonal
 - coordinates, 253
 - curves, 306
 - eigenfunctions, 427
 - eigenvectors, 137
 - functions, 42, 76, 427
 - random variables, 673
 - vectors, 148
 - Orthonormal, 137
 - Oscillations,
 - damped, 735
 - forced, 735
 - free, 735
 - small, 575
 - undamped, 735
 - Ostrogradsky, Michel, 144, 590
 - Output function, 8, 471
 - Overdamping, 738
- P**
- Parabolic equation. *See* Heat equation
 - Parallelogram law, 147
 - Parametric representation, 162
 - Paraxial approximation, 762
 - Parseval's relation for
 - energy signals, 463
 - power signals, 418
 - Partial
 - DE, 591
 - fractions, 506
 - sum, 339, 410
 - Particular solution, 5, 9, 132
 - Path. *See* Contour.
 - Pascal, Blaise, 638
 - Pendulum problem, 574
 - compound, 586
 - double, 577
 - Period, 296, 406
 - Periodic
 - boundary condition, 431
 - extension, 414
 - function, 296, 406
 - Sturm-Liouville system, 431
 - Permutation symbols,
 - Cartesian coordinates, 158, 218
 - rectilinear coordinates, 232
 - Phase, 748
 - plane, 129
 - spectrum, 462
 - Physical components, 239
 - Picard's theorem, 356
 - Piecewise continuous function, 409
 - Pochhammer symbol, 94
 - Point at infinity, 310
 - Pointwise convergence, 340, 409
 - Poisson
 - distribution, 645
 - equation, 209, 583
 - Poisson integral formula for
 - circular domain, 381

- half plane, 379, 628
 - Polar
 - coordinates, 608
 - form of complex numbers, 276
 - Pole
 - of order m , 353
 - simple, 354
 - Polygamma function, 71
 - Polynomials,
 - Chebyshev, 81
 - Hermite, 78
 - Laguerre, 80
 - Legendre, 41, 76
 - Positive direction, 322
 - Posterior density function, 671
 - Potential,
 - complex, 388
 - equation. *See* Laplace's equation
 - energy principle, 579
 - function, 183
 - Power
 - series, 34, 341
 - series method, 34
 - signal, 406, 461
 - spectral density, 711
 - Principal
 - axes, 120, 241
 - branch, 302
 - value, 277
 - Principle of
 - maximum likelihood, 671
 - minimum potential energy, 579
 - Probability
 - as relative frequency, 639
 - axioms of, 639
 - conditional, 655, 668
 - density function, 642
 - total, 669
 - Probability density functions, 642
 - binomial, 647
 - Cauchy, 655
 - chi-square, 665
 - conditional, 656
 - gamma, 647, 649
 - K , 684
 - negative exponential, 643
 - normal, 646, 647
 - Poisson, 645, 647
 - Rayleigh, 647, 648
 - Rician, 647
 - table of, 647
 - uniform, 92, 643, 647, 648
 - Product of
 - complex numbers, 274
 - random variables, 526, 683
 - vectors, 148-151
 - Projection of a vector, 149
 - Proper transformation, 219
 - Psi function. *See* Digamma function
 - Pure imaginary number, 274
- Q**
- Q -function, 682
 - Quotient
 - law, 224
 - of random variables, 685
- R**
- Radius of convergence, 342
 - Random process, 698
 - stationary, 700
 - Random variable,
 - continuous, 640
 - discrete, 640, 644
 - of mixed type, 644
 - products of, 526, 683
 - quotients of, 685
 - statistically independent, 672
 - sums of, 678, 688
 - Rayleigh distribution, 92, 647, 648
 - moments of, 652
 - Rayleigh
 - energy theorem, 463
 - principle, 568
 - quotient, 565
 - Rayleigh-Ritz method, 567
 - Real
 - axis, 275
 - part of, 274
 - Realization, 698
 - Reciprocal mapping, 388
 - Rectangle function, 64
 - Rectifiable curve, 324
 - Rectilinear coordinates, 218

- Recurrence formula, 38, 50
- Region, 282
- Regular
 - singular point, 44
 - Sturm-Liouville system, 425
- Relative frequency, 639
- Removable singularity, 353
- Residue, 357
- Residue theorem, 361
- Resonance, 616
- Ricci, G., 144
- Ricci's theorem, 257
- Rician distribution, 92, 681, 647
 - moments of, 753
- Rice-Nakagami distribution. *See* Rician distribution
- Riemann, Georg. F. B., 272
- Riemann
 - Christoffel tensor, 259
 - Stieltjes integral, 712
 - zeta function, 71
- Right-hand rule, 150
- RLC circuit, 509, 741
- Robin condition, 593
- Root-mean-square (rms) value, 715

- S**
- Sample space, 639
- Sampled function, 529
- Scalar, 145, 216
 - field, 170
 - multiplication, 148
 - triple product, 151
- Schwarz inequality, 137
- Self-adjoint equation, 425
- Separation of variables,
 - ODEs, 7
 - PDEs, 594
- Sequency domain, 540
- Series,
 - geometric, 340
 - of Bessel functions, 438
 - convergence of, 339, 342, 409
 - of eigenfunctions, 428, 611
 - Fourier, 405
 - Laurent, 346
 - Maclaurin, 344
 - partial sum of, 339, 410
 - power, 341
 - Taylor, 344
- Sifting property, 66
- Signal-to-noise ratio, 92, 750
- Signum function, 63
- Simple
 - curve, 321
 - harmonic function, 601, 735
 - pole, 353
 - zero, 352
- Simply connected, 183
- Sinc function, 65
- Sine,
 - integral representation, 457
 - inverse, 304
 - series, 412
 - transform, 460
- Singular
 - matrix, 114
 - Sturm-Liouville system, 432
- Singular point, 36, 292, 352
 - branch, 302
 - essential, 353
 - irregular, 44
 - isolated, 353
 - pole, 353
 - regular, 44
 - removable, 353
- Sink, 391
- Skew-symmetric tensor, 225
- Smooth
 - curve, 180, 321
 - surface, 185
- Solution, 4
 - general, 5, 9, 591
 - homogeneous, 9
 - particular, 5, 9
 - steady-state, 596
 - trivial, 9
- Source, 391
- Spectral density, 711
 - cross, 721
- Spectrum,
 - line, 419
 - magnitude, 462
 - phase, 462

- Spherical Bessel functions, 92
 Spherical coordinates, 201, 268
 Square
 -integrable, 406
 matrix, 110
 wave, 410
 Square-law device, 661, 716
 Stable system, 129, 511
 asymptotically, 130
 Stagnation point, 388
 Standard deviation, 651
 Standing waves, 601
 Stationary,
 function, 549
 increments, 707
 in the wide sense, 702
 random process, 700
 value, 549
 Steady-state solution, 596
 Step function, 63
 Stereographic projection, 310
 Stirling's formula, 691
 Stochastic variable. *See* Random variable
 Stokes, George G., 144
 Stokes's theorem, 204
 Stream function, 388
 Streamline, 387
 Structure function, 707
 Sturm, Jacques C. F., 404
 Sturm-Liouville system, 425, 431, 432, 567
 Sum of random variables, 678, 688
 Summation convention, 156
 Superposition principle, 18, 595
 Surface,
 area, 189
 integral, 185
 normal to, 186
 Surge, 728
 Symmetric
 matrix, 113, 119
 tensor, 225
 System
 function, 472
 linear, 471
 of DEs, 121
- T**
 Table of
 covariance functions, 714
 distributions, 647
 Fourier transforms, 464
 Laplace transforms, 501
 Hankel transforms, 517
 Mellin transforms, 522
 power spectral densities, 714
 Z-transforms, 535
 Tangent vector, 162
 Taylor
 series, 344
 theorem, 344
 Tensor,
 components, 156
 contravariant, 222
 covariant, 222
 definition of, 221
 metric, 228, 248
 mixed, 222
 notation, 156, 216
 physical components, 239
 transformation laws, 221
 Thermal
 conductivity, 599
 diffusivity, 593
 Time
 average, 703
 invariant, 716
 Torsion of a curve, 165
 Total probability, 669
 Trajectories, 129
 Transfer function, 472, 479, 481, 510
 Transient solution, 597
 Transpose of a matrix, 113
 Traveling waves, 604
 Triangle
 function, 64
 inequality, 137
 Trigonometric
 form of complex numbers, 277
 functions, complex, 297
 series. *See* Fourier series
 Triple
 integral, 200
 product, 151, 235

Trivial solution, 9
 Two-dimensional
 distribution, 665
 Fourier transform, 477

U

Uncorrelated random variables, 673
 Undamped motion, 735
 Underdamping, 738
 Undetermined coefficients, 24
 Uniform
 convergence, 340
 distribution, 92, 643, 647, 648
 Unit
 binormal vector, 164
 circle, 363
 impulse response. *See* Impulse
 response function
 normal vector, 164
 step function. *See* step function
 tangent vector, 163
 vector, 145
 Unitary space, 136
 Unstable system, 130, 512
 marginally, 512

V

Variance of a distribution, 651
 Variation of parameters, 9, 28, 132
 Vector, 145, 216
 components, 146
 cross product, 150, 235
 differential operator, 177
 dot product, 148, 235
 field, 173
 magnitude, 145
 space, 133
 triple product, 151, 235
 unit, 145
 zero, 145
 Velocity,
 complex, 387
 potential, 388
 vector, 166
 Vibrating
 membrane, 583
 string, 580, 600

Volume integral, 200
 Vortex flow, 391

W

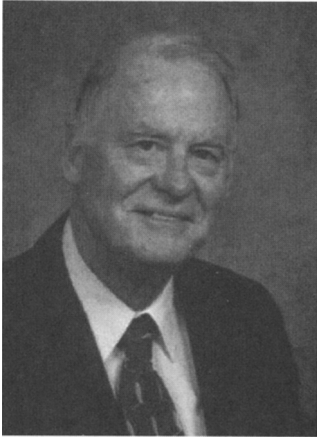
Walsh
 functions, 539
 transform, 538
 Wave equation, 582, 600
 d'Alembert solution of, 602
 Wavelet, 487
 transform, 491
 Weierstrass, Karl, 62, 272
 Weierstrass M -test, 341
 Weighted residual, 568
 Weighting function, 425
 White noise, 720
 Wide-sense stationary, 702
 Wiener-Khinchin theorem, 713
 Work, 182, 184
 Wronskian, 19, 122

Y

Young's modulus, 581

Z

Zernike polynomials, 773
 aperture filter functions, 777
 modal expansion, 776
 table of, 776
 Zero
 of analytic function, 352
 crossings, 726
 vector, 145
 Zeta function. *See* Riemann zeta
 function
 Z-transform, 533
 properties of, 535
 table of, 535



Larry C. Andrews is a professor of mathematics at the University of Central Florida and an associate member of the School of Optics/CREOL. He is also an associate member of the Florida Space Institute (FSI). Previously, he held a faculty position at Tri-State University and was a staff mathematician with the Magnavox Company, antisubmarine warfare (ASW) operation. He received a doctoral degree in theoretical mechanics in 1970 from Michigan State University. Dr. Andrews has been an active researcher in optical wave propagation through random media for more than 20 years and is the author or co-author of 10 textbooks on differential equations, boundary value problems, special functions, integral transforms, and wave propagation through random media. Along with wave

propagation through random media, his research interests include special functions, random variables, atmospheric turbulence, and signal processing.



Ronald L. Phillips is Director of the Florida Space Institute (FSI) and a professor in the Department of Electrical and Computer Engineering at the University of Central Florida. Dr. Phillips is also a member of the Department of Mathematics and an associate member of the Center for Research and Education in Optics and Lasers (CREOL). He has held positions on the faculties at Arizona State University and the University of California, San Diego. He received a doctoral degree in Electrical Engineering in 1970 from Arizona State University. Dr. Phillips has been an active researcher in wave propagation through random media for more than 22 years. He was awarded a Senior NATO

Postdoctoral Fellow in 1977 and the American Society for Engineering Education 1983 Medal for outstanding contributions in research. Dr. Phillips is co-author of two textbooks on wave propagation through random media and, in addition to optical wave propagation, his research interests include optical communications and imaging through atmospheric turbulence.