

Chapter 5

A Mathematical–Observational Duality

5.1 The End of Intelligibility

When discussing the enormity of the transformation wrought by Galileo and Newton, Kline states, “What science has done, then, is to sacrifice physical intelligibility for the sake of mathematical description and mathematical prediction.” [Kline, 1985] Sacrificing physical intelligibility does not involve an abandonment of knowledge; on the contrary, it involves the recognition that everyday human categories concerning Nature—those that arise from ordinary interaction with the physical world, such as pushing and pulling—are, at best, only suitable for describing simple phenomenal relations. Kline writes,

The insurgent seventeenth century found a qualitative world whose study was aided by mathematical abstractions. It bequeathed a mathematical, quantitative world that subsumed under its mathematical laws the concreteness of the physical world. In Newton’s time and for two hundred years afterwards, physicists spoke of the action of gravity as ‘action at a distance,’ a meaningless phrase that was accepted as a substitute for explaining the physical mechanism, much as we speak of spirits or ghosts to explain unseen phenomena. [Kline, 1985]

Kline’s point is twofold. First, the transformation to a mathematical world was accomplished before the end of the Seventeenth Century. Second, for two hundred years afterwards many scientists refused to accept this transformation—and many today still do not.

5.2 Quantum Mechanics

The development of quantum mechanics during the first third of the Twentieth Century compelled scientists to confront the epistemological issues lurking within Newton’s *Hypotheses non fingo*, as it applies to causality/determinism and to the structure and validation of scientific theories. This section describes some basic aspects of quantum theory that foster a deeper understanding of what it means for knowledge to be framed as a mathematical-observational duality and then discusses epistemological implications.

5.2.1 The Bohr atom

Up until shortly after the beginning of the Nineteenth Century, Newton's corpuscular theory of light, which claimed that light consisted of tiny particles, was widely accepted. Then, around 1803, Thomas Young performed his famous double-slit experiment in which light from a point source emanated in the direction of a barrier with two holes (called "slits"), passed through the slits, and was captured on a flat detector (Fig. 5.1). The light arriving on the detector was distributed in a manner consistent with wave interference from the light passing through the two slits, not as one would expect if particles were passing through the slits. Although not accepted at first, Young's wave theory became predominant in the Nineteenth Century.

In 1900, based on his study of blackbody radiation, Max Planck proposed that light and other electromagnetic waves are emitted in discrete packets (*quanta*) of energy that can only take on certain discrete values. These values are multiples of a constant h , now called *Planck's constant*. Energy radiated from a blackbody must be a multiple of hf , f being the frequency of the radiation.

In 1905, in the paper that earned him the Nobel Prize in 1921, Einstein went further by not just claiming emission in discrete packets, but that light is composed of discrete packets. He did this by considering the *photoelectric effect*, discovered in 1887 by Heinrich Hertz, which refers to the ejection of electrons by metals when exposed to light. Behavior that he observed regarding the ejected electrons appeared inconsistent with the view that light is a wave phenomenon.

Regardless of brightness, only light above a certain frequency prompts electrons to emit. As the frequency increases, the maximum kinetic energy of the ejected electrons increases proportionally with the frequency of the light, but does not vary with the intensity of the light, which would accord with wave theory. Moreover, the electrons are emitted almost simultaneously with the arrival of the light. Einstein explained the behavior of the emissions by assuming light to be made of individual particles, later called *photons*. Each photon possesses a quantum of energy $E = hf$. Hence, argued Einstein, it is not simply the emission of energy that is quantized, but that energy itself is quantized.

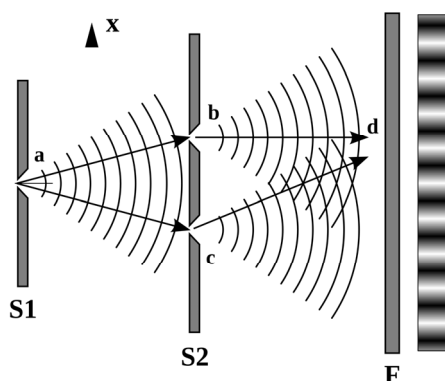


Figure 5.1 Young's double-slit experiment [Fermilab Today, 2008].

A second discrete phenomenon, discovered in the Nineteenth Century, concerned atomic emission spectra. When solids, liquids, and dense gases are heated to high temperatures, for instance, as occurs when electricity is passed through a light filament, light possessing a continuous spectrum is emitted. However, when energy is supplied to gas atoms under low pressure, the atoms emit light consisting of only discrete frequencies and these form a discrete atomic emission spectrum (Fig. 5.2).

In 1897, J. J. Thomson proposed a model of the atom in which tiny negatively charged electrons float in a “pudding” of positive charge. In 1911, Ernest Rutherford shot high-velocity alpha particles (helium nuclei) into gold foil and captured the locations of the alpha particles on a fluorescent screen after they had passed through the gold foil. Most of the alpha particles passed through with very little deflection, as might be expected given the Thomson model; however, some deviated substantially and a small number bounced back. Rutherford hypothesized that the atom had a small dense positively charged nucleus at its center with negatively charged electrons orbiting around it. Although this planetary model was consistent with the charges and the behavior of the alpha particles in his experiment, it had problems. In particular, an electron circling a nucleus should be continually sapped of its energy and thus rapidly spiral into the nucleus. Moreover, the model could not explain discrete atomic emission lines.

To correct some of the defects in the Rutherford model, in 1913, Niels Bohr hypothesized that electrons orbit the nucleus of an atom at discrete distances, the actual distances depending on the element (Fig. 5.3) Electrons closer to the nucleus have lower energy than those further away. An electron must occupy definite energy levels, known as *quantum states*. It can jump to a different level without passing through intermediate levels, a so-called *quantum jump*. If light with the right energy encounters an atom, then the light will be absorbed, the atom’s electrons will be excited, and they will rise to higher energy states. In the other direction, when an electron jumps from a higher energy orbit to a lower one, it emits a photon whose energy equals the difference between the energy levels of the orbits. The discrete jumps fit neatly with the discrete spectral lines.

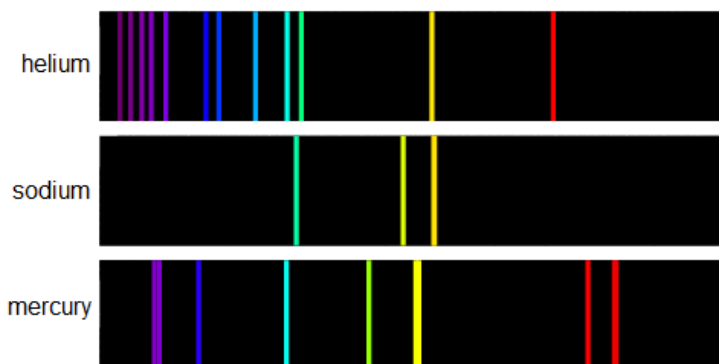


Figure 5.2 Atomic emission spectra.

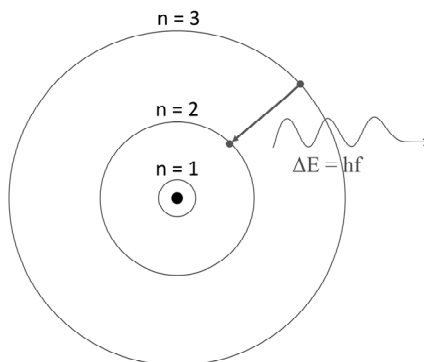


Figure 5.3 Bohr atomic model [Wikipedia, 2007].

5.2.2 Wave–particle duality

While the Bohr model predicts empirical observations better than the Rutherford model, quantum jumps are not in accord with our ordinary experience of continuity: an electron never occupies space between its original level and the one to which it jumps. Furthermore, is light a particle or a wave? Our ordinary experience seems to say that it must be one or the other, but not both. But suddenly it appears that light behaves as both a particle and a wave, depending on the experiment. Thus, physicists are confronted with a wave–particle duality, a notion that defies our ordinary categories of the understanding.

In 1924, Louis de Broglie argued that wave–particle duality is characteristic of both radiation and all particles of matter, not just light. Whereas Planck and Einstein had demonstrated that what was thought to be waves act like particles, de Broglie asserted that what was thought to be particles act like waves. In the case of electrons, a wave-like character implies that interference can only be avoided by occupying orbits at certain distances from the nucleus, in accordance with the Bohr atomic model. De Broglie’s wave–particle duality theory was later supported when wave-like interference patterns were observed when electrons were passed through a double-slit experiment.

We consider the double-slit experiment more closely. It is possible to generate light of such low intensity that the experimenter can keep track of individual photons and record hits on the detector as they build up. It turns out that each individual photon falls randomly on the detector; however, after a large number of photons have arrived, a wave pattern emerges. What then is the path of an individual photon? Which slit does it go through, or does it go through both? Are these questions even meaningful? All that is known is that probabilities can be assigned to regions in which a photon might hit, these being consistent with the wave pattern. Various experiments have been performed and sundry observations have been made. What seems to be safe to say is that, from the perspective of ordinary understanding, strange phenomena have been observed.

To illustrate wave–particle behavior associated with a double-slit experiment, we consider an experiment performed by a group led by Akira Tonomura. Single electrons are emitted one by one from the source in an electron microscope. They pass through a device called an “electron biprism,” which consists of two parallel plates with a fine filament at the center (each side of which corresponds to a slit) and they are individually observed as particles on a detector. Parts (a) through (e) of Fig. 5.4 show increasing numbers of electrons on the detector: 11, 200, 6000, 40,000, and 140,000. With a small number of electrons, the pattern appears completely random; however, as the number of electrons increases the interference pattern becomes increasingly visible, even though the electrons are emitted individually. Are the electrons waves or particles?

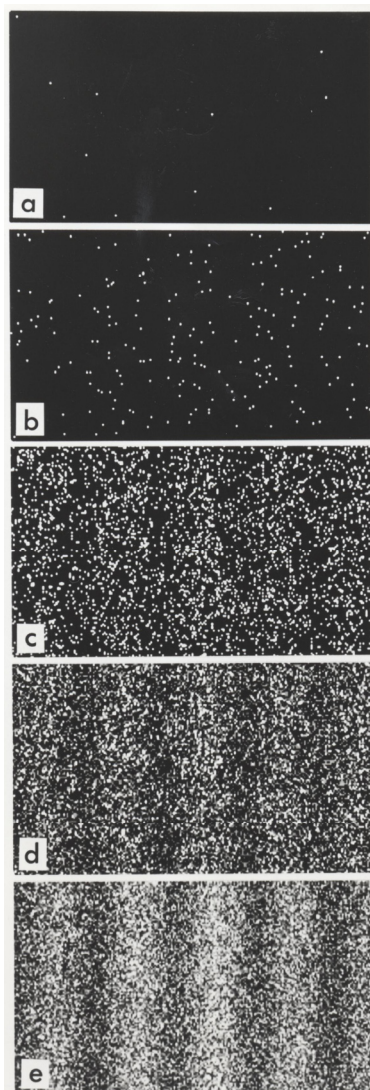


Figure 5.4 Electron waves in the double-slit experiment [Wikipedia, 2012].

Regarding the wave–particle behavior observed in double-slit experiments, in his *Lectures on Physics*, Richard Feynman writes,

In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by “explaining” how it works. We will just *tell* you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics. [Feynman, 1964]

Once an individual electron hits the detector, its position is known exactly, but before then its position can only be described probabilistically. The behavior of an electron is governed by Schrödinger’s wave equation (for the mathematical form of which we refer the interested reader to the abundant literature). Max Born showed that the square of the wave function is a probability density governing particle position. In principle, Schrödinger’s equation applies to all non-relativistic matter; however, only for small systems are the wavelengths observable and significant. Schrödinger solved for the exact solutions of the wave equation for the hydrogen atom. The results match the known energy levels of the atom. Figure 5.5 shows probability density plots for the hydrogen atom orbitals. The plots are two-dimensional slices; the actual densities are three-dimensional. Given that an electron is in an orbital, the probability of finding the electron in any region of the orbital is the probability of that region.

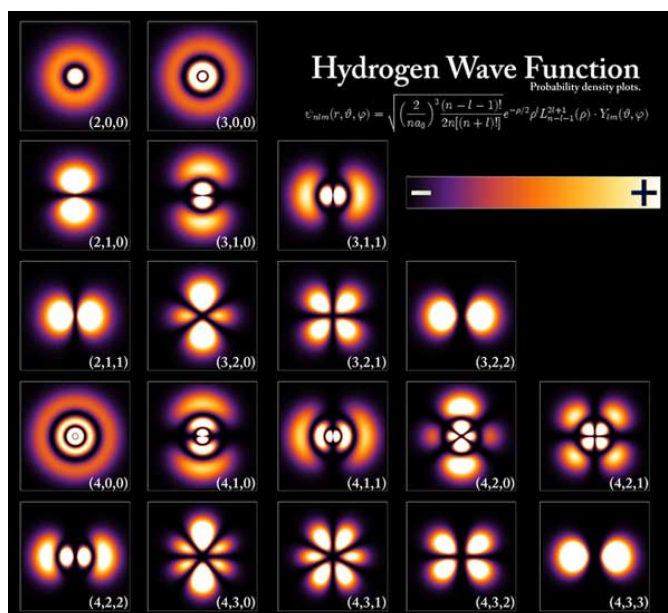


Figure 5.5 Hydrogen atom orbitals [Sevencolors.org, 2009].

5.2.3 The uncertainty principle

In 1927, Werner Heisenberg stated the famous uncertainty principle with which his name is often associated:

At the instant of time when the position is determined, that is, at the instant when the photon is scattered by the electron, the electron undergoes a discontinuous change in momentum. This change is the greater the smaller the wavelength of the light employed, i.e., the more exact the determination of the position. At the instant at which the position of the electron is known, its momentum therefore can be known only up to magnitudes which correspond to that discontinuous change; thus, the more precisely the position is determined, the less precisely the momentum is known, and conversely. [Heisenberg, 2006]

Heisenberg originally conceived the idea by considering the measurement of a particle's position and velocity using an optical microscope. Light hits the particle and is reflected. When the light photons hit a sub-atomic particle, it moves. The position is accurately measured but the velocity of the particle is affected. Hence, the position is obtained but knowledge pertaining to the velocity is lost. Based upon this thinking, Heisenberg proposed that the certainty with which we know the location of a particle is inversely related to the certainty with which we know its momentum.

The uncertainty principle is often written as $\Delta x \Delta p \geq h/4\pi$, where Δx and Δp denote the uncertainties in position and momentum, respectively, and h is Planck's constant. More precisely, it takes the form $\sigma_x \sigma_p \geq h/4\pi$, where σ_x and σ_p denote the standard deviations of the position and momentum, respectively. Whereas Heisenberg originally thought of the uncertainty principle as due to the measurement process, it arises as a consequence of the quantum wave nature of the electron. Consequently, it is a fundamental physical property, not a statement concerning measurement technology.

According to the uncertainty principle, a particle does not possess specific position and velocity; instead, these are known probabilistically and there is an intrinsic limitation of the accuracy with which the system composed of the position and momentum can be known. Physical laws can only provide probabilistic descriptions up to the limit allowed by the uncertainty principle. Epistemologically, this differs radically from the basic deterministic principle of classical physics in which the state of a system can be precisely determined and, once this is determined, future states can be predicted precisely by the laws.

5.3 Epistemological Reflections on Quantum Theory

Quantum theory is inconsistent with many commonplace assumptions: continuity, causality, determinism, a particle having a unique position, and the distinction between particles and waves. Thus, it is not surprising that the theory

provoked much debate as to its meaning, its status as a physical theory, and its implications for epistemology.

5.3.1 The Copenhagen interpretation

Prior to the measurement of its position on the detector, an electron has no definite position, at least insofar as physics is concerned, there being only a probability distribution characterizing the likelihood of its position, but once detected, it has a definite position. How is this to be interpreted? The view taken by Bohr and Heisenberg is that once a particle is measured, the probability of its being detected elsewhere becomes zero. Prior to detection, the particle's position is inherently random. The randomness disappears upon interaction with a measuring device. Bohr believed that there is no precise way to define the exact point at which this so-called *wave function collapse* occurs. Hence, there is no deep quantum reality, no actual world of electrons and photons. Quantum mechanics provides a formalism that we can use to predict and manipulate events. There is no knowledge beyond that. However, once the measurements are made, these behave in the classical manner and should be describable in classical language. On account of Bohr's laboratory being in Copenhagen, this perspective is known as the *Copenhagen interpretation*.

From the human perspective, the theory views Nature as intrinsically random and somehow interdependent with human observation. One thinks of Berkeley (*esse est percipi*). When observed, an electron has a position; when not being observed it does not. And, according to the uncertainty principle, if it is observed with perfect precision, then its momentum, which means its velocity, is totally unknown.

Einstein was uncomfortable with this interpretation. There can be no proof that there are not hidden variables whose discovery would eliminate randomness. Perhaps quantum theory is incomplete. This would agree with Laplace's view that the randomness we observe is always due to ignorance. The argument cannot be decided beforehand, that is, before the actual discovery of the variables, so that they are no longer hidden. Beyond that, Einstein believed that science has to be deterministic because he believed reality is deterministic. Referring to the Seventeenth Century philosopher Baruch Spinoza, Einstein wrote, "He was utterly convinced of the causal dependence of all phenomena, at a time when the success accompanying efforts to achieve a knowledge of the causal relationship of natural phenomena was still quite modest." [Einstein, 1982] Thus, Einstein is taking a metaphysical position in agreement with Spinoza.

Although there are other interpretations of quantum theory, it appears that the Copenhagen interpretation is held by the majority of physicists. This is consistent with Newton's *Hypotheses non fingo*, although one would be rash to conclude that Newton would agree with the extension of his dictum to the Copenhagen interpretation. In any event, it is a minimalist view and consistent with maintaining a demarcation between science and metaphysics.

5.3.2 Knowledge depends on the questions asked

As one might expect from the originator of the uncertainty principle, Heisenberg puts great emphasis on the interaction between the scientist and Nature. He writes, “Natural science does not simply describe and explain Nature; it is part of the interplay between nature and ourselves.” The key to that interplay is the manner in which we probe Nature. In Heisenberg’s words, “What we observe is not Nature itself, but Nature exposed to our method of questioning.” Think of the uncertainty principle. Does the question concern the position or the momentum? Heisenberg says that we must choose where to put our focus: “We decide, by our selection of the type of observation employed, which aspects of nature are to be determined and which are to be blurred.” [Heisenberg, 1977a]

Since the knowledge gained depends on the questions asked, the mathematical system, which constitutes the frame of thinking, is in some sense determinative of the kind of knowledge to be gained because the questions must lead to answers that can be formulated in the language of the system. Thus, depending on the mathematical system chosen, the same phenomena may be modeled (thought about) in different ways. Heisenberg considers this idea to be the most important concept arising from quantum theory:

The most important new result of nuclear physics was the recognition of the possibility of applying quite different types of natural laws, without contradiction, to one and the same physical event. This is due to the fact that within a system of laws which are based on certain fundamental ideas only certain quite definite ways of asking questions make sense, and thus, that such a system is separated from others which allow different questions to be put. [Heisenberg, 1977b]

Questions presuppose answers and scientific answers are quantitative. They involve measurement. The uncertainty principle raises the following question: Does a property that cannot be measured exist? According to Percy Bridgman,

On careful examination the physicist finds that, in the sense in which he uses language, no meaning at all can be attached to a physical concept which cannot ultimately be described in terms of some sort of measurement. A body has position only in so far as its position can be measured; if a position cannot in principle be measured, the concept of position applied to the body is meaningless, or in other words, a position of the body does not exist. Hence if both the position and velocity of the electron cannot in principle be measured, the electron cannot have the same position and velocity; position and velocity as expressions of properties which an electron can simultaneously have are meaningless. To carry the paradox one step further, by choosing whether I shall measure the position or the velocity of the electron, I thereby determine whether the electron has position or velocity. The physical properties of

the electron are not inherent in it, but involve also the choice of the observer. [Bridgman, 1950]

It has long been known that science is inextricably tied to technology because the capacity to measure depends directly on the instrumentation available, but quantum theory goes beyond that by saying that certain measurements are intrinsically impossible and therefore the impossibility of measurement cannot be overcome by improved technology.

5.3.3 Nature is absurd

Given that scientific knowledge depends on the questions asked, which are in turn limited by the mathematical apparatus and the measurement process, what then is the relation between scientific knowledge and Nature? On this most fundamental point, Bohr takes a Kantian position: “It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we say about Nature.”

For Bacon, the essence of a phenomenon pertains to its metaphysical form, which constitutes a deeper reality than the empirical observation and would have to be where meaning resides. Bohr dismisses any hope for meaning:

A subsequent measurement to a certain degree deprives the information given by a previous experiment of its significance for predicting the future course of the phenomena. Obviously, these facts not only set a limit to the *extent* of the information obtainable by measurements, but they also set a limit to the *meaning* we may attribute to such information. We meet here in a new light the old truth that in our description of Nature the purpose is not to disclose the real essence of the phenomena (i.e., the quantum character of their ultimate constitution) but only to track down, as far as possible, relations between the manifold aspects of our experience. [Bohr, 2012]

Indeed, it is an “old truth”—in Galileo, Newton, and Kant.

For Kant there is a deeper reality, the noumena, but this is not accessible to the categories of the understanding, which apply to phenomena. Here Bohr parts with Kant because Bohr’s description of Nature is not limited to the categories of the understanding; indeed, it is precisely the ordinary human understandings about Nature that quantum mechanics rejects.

What we can say about Nature depends on what we can observe and what mathematical tools can be brought to bear. As Newton’s desire to quantitatively express mechanical concepts led him to develop the calculus, the probabilistic nature of quantum events in space and time helped spur the rapid development of the theory of random processes in the 1930s and 1940s. The formulation of quantum theory in terms of operators depends on the theory of Hilbert spaces, which illustrates the dependency of science on the language of mathematics. There is the famous example of Einstein approaching David Hilbert for help in

formulating the general theory of relativity and Hilbert suggesting Riemannian geometry as an appropriate language.

Did quantum theory fundamentally advance the epistemology of the Seventeenth Century, which, as stated by Kline, “bequeathed a mathematical, quantitative world that subsumed under its mathematical laws the concreteness of the physical world?” Perhaps not theoretically! But practically it did. One could no longer depend on the language of ordinary experience, such as “wave” and “particle,” to formulate laws. One could no longer depend on using everyday models such as billiard balls banging into each other to explain the theory. Galileo had dismissed explanation as science in principle. Quantum mechanics left no doubt that Nature cannot be described in mental pictures.

In the *Mysterious Universe*, James Jeans writes,

The final truth about phenomena resides in the mathematical description of it; so long as there is no imperfection in this, our knowledge is complete. We go beyond the mathematical formula at our own risk; we may find a [nonmathematical] model or picture that helps us to understand it, but we have no right to expect this, and our failure to find such a model or picture need not indicate that either our reasoning or our knowledge is at fault. [Jeans, 1930]

Non-mathematical reasoning may be useful for the scientist in exploratory thinking, but scientific knowledge is constituted in a mathematical model. One might use a metaphor of observers holding lights on approaching trains to make an intuitive point concerning relativity, but the scientific theory lies properly within the equations. Any attempt to force a non-mathematical understanding creates the risk of having a diminished (or erroneous) scientific theory because it substitutes readily understandable and often convincing descriptions in place of strict scientific knowledge, which must take a mathematical form.

With all of this mathematics, where is the concreteness of the physical world? Indeed, is there something concrete? If we cannot express it, then is there an “it” to express? Jeans writes,

A mathematical formula can never tell us what a thing is, but only how it behaves; it can only specify an object through its properties. And these are unlikely to coincide *in toto* with the properties of any single macroscopic object of our everyday life.... We need no longer discuss whether light consists of particles or waves; we know all there is to be known about it if we have found a mathematical formula which accurately describes its behavior, and we can think of it as either particles or waves according to our mood and the convenience of the moment. [Jeans, 1930]

There is behavior apprehended as measurements. These are abstracted as variables in a mathematical system and comprise the elements related by the

mathematics. That is it. Concreteness is a will-o'-the-wisp. Not only is there an unbridgeable chasm between the phenomenal and noumenal worlds, there is also a huge gulf between human understanding and the phenomena.

Schrödinger states the matter metaphorically:

As our mental eye penetrates into smaller and smaller distances and shorter and shorter times, we find nature behaving so entirely differently from what we observe in visible and palpable bodies of our surrounding that no model shaped after our large-scale experiences can ever be 'true'. A completely satisfactory model of this type is not only practically inaccessible, but not even thinkable. Or, to be precise, we can, of course, think it, but however we think it, it is wrong; not perhaps quite as meaningless as a 'triangular circle', but much more so than a 'winged lion'. [Schrödinger, 2004]

Where does this leave us in our relationship with Nature? Beginning a lecture series on quantum electrodynamics to an audience of non-specialists, Richard Feynman is unequivocal:

What I am going to tell you about is what we teach our physics students in the third or fourth year of graduate school—and you think I'm going to explain it to you so you can understand it? No, you're not going to be able to understand it.... You see, my physics students don't understand it either. That is because I don't understand it. Nobody does.... It is whether or not the theory gives predictions that agree with experiment. It is not a question of whether a theory is philosophically delightful, or easy to understand, or perfectly reasonable from the point of view of common sense. The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you can accept Nature as she is—absurd. [Feynman, 1985]

Nature qua Nature is not absurd. Nature qua the human categories of the understanding is absurd. Would it not be presumptuous to suppose otherwise? A mathematical theory is intelligible because it is a product of the human intellect; Nature is not a product of the human intellect.

5.4 The Structure of Scientific Knowledge

Feynman's statement posits two definitive assumptions underlying scientific knowledge: (1) understanding in the form of intelligibility is neither necessary nor sufficient for scientific knowledge, and (2) the sole criterion for the validity ("truth") of a scientific theory is concordance between predictions derived from the theory and corresponding observations.

Everything begins with an experiment designed to answer questions in the mind of the scientist. The product of an experiment is a set of measurements that

form the data of sensibility, the empirical (as opposed to a rational) basis for knowledge. In themselves, measurements do not constitute scientific knowledge. They must be integrated into a conceptual system. Scientific knowledge is constituted via synthesis of the observed measurements. These are related to variables and relations among the variables. Modern science is based on the integration of two fundamental principles: (1) the design of experiments under constrained circumstances to extract specifically desired information; and (2) the mathematical formulation of knowledge. The two principles arise from the two sides of the scientific problem, the source of knowledge and the representation of knowledge in the knower.

Scientific knowledge necessarily takes the form of mathematics for four reasons:

1. Scientific knowledge is based on quantitative measurements, be they logical or numeric.
2. Scientific knowledge concerns relations, and mathematics provides the formal structure for relations.
3. The validity of a scientific theory depends on predictions, and this requires a quantitative structure from which to generate predictions and a theory of probability in which the goodness of predictions can be quantified.
4. Mathematics provides a formal language in which both the constituting theory and the experimental protocols for prediction are inter-subjective, once the underlying mathematical representation of the theory is agreed upon.

Regarding the last requirement, Karl Popper (1902–1994) writes, “The objectivity of scientific statements lies in the fact that they can be inter-subjectively tested.” [Popper, 1959] Inter-subjectivity demands that scientific knowledge not depend on reason, except within the strict rules of mathematics and logic; otherwise, philosophical theories like Marxism could legitimately claim to be science. This would be “cult science,” open only to those who claim to understand empty phrases such as “dialectical materialism.”

There is much more to a model than the defining relations, that is, the general principles of the model. A great power of the scientific epistemology lies in the deducibility of logically necessary relations from the defining relations—the *hypothetico-deductive method*. This deduction can reveal critical relations not at once apparent in the defining relations. A full mathematical model consists of the defining relations and all relations logically deduced from these. The knowledge constituted by the derived relations is implicit in the defining structure but only becomes apparent when derived explicitly.

A mathematical model alone does not constitute a scientific theory; the model must be related to phenomena, that is, the formal mathematical system must be related to the empirical ground of science. Validation of a system requires that it be tied to observations by rules that relate not necessarily to its defining relations but to conclusions logically deduced from the defining relations. There must be a formal protocol for testing the theory by checking

measurable consequences of the theory. Bridgman observed that the relational rules involve the description of physical operations and called them *operational definitions*.

The operational definitions are an intrinsic part of a scientific theory, for without them there would be no connection between the mathematics and observation, between the conceptual system and the experiments. The conceptual system must have consequences that can be checked via their relation to sensory observations. There must be a defined procedure for relating the consequences of the equations to quantifiable observations, such as the compression of a spring or the distribution of electrons on a detector.

A scientific theory consists of two parts:

1. A mathematical model composed of symbols (variables and relations between the variables).
2. A set of operational definitions that relate the symbols in the model and measurements of corresponding physical events.

In addition, two requirements must be met to have a validated scientific theory:

3. There must be validating data, that is, a set of future quantitative predictions derived from the theory and corresponding measurements.
4. A statistical analysis that supports acceptance of the theory, that is, supports the concordance of the predictions with the physical measurements—including the mathematical theory justifying application of the statistical methods.

The fourth requirement means that one cannot apply a statistical technique unless there is solid theory demonstrating the validity and specificity of the conclusions drawn relating the predictions and measurements, and there is theoretical justification for applying the statistical technique under the current conditions. For instance, if the statistical theory requires that the data come from a normal distribution, then there must be evidence that an assumption of normality, while not necessarily guaranteed, is at most only weakly violated. One might apply a hypothesis test and show that the data do not support rejection of the normality assumption.

5.5 Scientific “Truth”

For Plato, true knowledge involves certainty and resides in the deeper reality of the forms, not in the shadow world of empirical observations, where uncertainty prevails. While dismissing the deeper reality as a fiction, Hume agrees that knowledge gained via the senses is inherently uncertain. This does not leave us with a categorical absence of knowledge, nor does it render the notion of truth meaningless. On the contrary, taking expectation as the ground of scientific knowledge leads to the basis of scientific truth. Predictive relations characterize model validity and are necessary for scientific knowledge. Truth is determined by

concordance of the predictive relations with future observations corresponding to the predictions. Scientific truth relates to the predictive capacity of a scientific theory. Scientific knowledge is about the future. Past observations may lead to discovery of a theory but the theory must predict the future.

Reichenbach writes,

If the abstract relations are general truths, they hold not only for the observations made, but also for observations not yet made; they include not only an account of past experiences, but also predictions of future experiences. That is the addition which reason makes to knowledge. Observation informs us about the past and the present, reason foretells the future. [Reichenbach, 1971]

Foretelling the future is the crux. A model may fit existing data, but the model must incorporate mathematical machinery that makes it predictive across time to be scientifically valid.

Prediction is not certitude. Instead of causality, science involves conditional distributions that describe the probability of a *target* random variable Y given the values of a set of *predictor* random variables, X_1, X_2, \dots, X_m . The target measures some process, and it has a probability distribution quantifying its behavior. The predictor variables possess the quality of causes in that their outcomes condition the behavior of the target, in analogy to causes determining an effect, but they do so in a probabilistic manner. Specifically, the original probability distribution of the target Y is altered depending on the outcomes of the predictors X_1, X_2, \dots, X_m . In particular, given values of the predictor random variables, the best prediction (relative to mean-square error) of Y is its conditional expectation, meaning its expectation conditioned on the values of X_1, X_2, \dots, X_m .

Causality is replaced by conditioning. Statements concerning conditional prediction can be validated via experimentation. The meaning of a statement can be defined within the framework of probability theory, and its relation to measurable phenomena can be mathematically characterized within the theory of statistics. If the predictor variables are antecedent to the variable to be predicted, then we have forward prediction. The terms “cause” and “effect” never appear.

The general epistemological perspective does not specify how it is to be applied in particular settings. According to Einstein,

In order that thinking might not degenerate into ‘metaphysics,’ or into empty talk, it is only necessary that enough propositions of the conceptual system be firmly enough connected with sensory experiences and that the conceptual system, in view of its task of ordering and surveying sense experience, should show as much unity and parsimony as possible. Beyond that, however, the system is (as regards logic) a free play with symbols according to (logically) arbitrarily given rules of the game. [Einstein, 1944b]

The model (conceptual system) is a creation of the imagination, in accordance with the rules of the game. The manner of this creation is not part of the scientific theory. The classical manner is that the scientist combines an appreciation of the problem with reflections upon relevant phenomena and, based on mathematical knowledge, creates a model. As Einstein states, this creation is free except that it must conform to the rules of the mathematical game.

Epistemologically more problematic is that Einstein's prescription does not lead to a unique, absolute truth because validation is a process and the "truth" of the theory is relative to that process. Indeed, what is meant by "enough propositions" being "firmly enough connected with sensory experiences?" How many propositions? How firmly? The model must be connected to observations but the specification of this connection in a given circumstance is left open. This specification constitutes an epistemological requirement that must be addressed in mathematical statements. Absent such a specification, a purported scientific theory is meaningless. Different people may set different requirements, so that one may accept the theory as valid and the other may not.

A scientific theory is incomplete without a formal specification of achievable measurements that can be compared to predictions derived from the conceptual theory and the manner in which the measurements are to be compared to the conceptual system, in particular, validity criteria and the mathematical properties of those criteria as applied in different circumstances. The validity of a theory is relative to this specification, but what is not at issue is the necessity of a set of relations tying the conceptual system to operational measurements. A scientific theory is inter-subjective, but the epistemological criteria underlying a particular validation are open to debate. Once the validation requirements are specified, the mathematical model (conceptual system) is valid relative to the validation criteria and to the degree that the requirements are satisfied, that is, to the degree that predictions demanded by the validation protocol and resulting from the mathematical model agree with experimental observations.

Reichenbach states, "Scientific philosophy has constructed a *functional* conception of knowledge, which regards knowledge as an instrument of prediction and for which sense observation is the only admissible criterion of nonempty truth." [Reichenbach, 1971]

Scientific knowledge is worldly knowledge in the sense that it points into the future by making predictions about events that have yet to take place. Scientific knowledge is contingent, always awaiting the possibility of its invalidation. Its truth or falsity lies in the verity of its predictions and, since these predictions depend upon the outcomes of experiments, ultimately the validity of scientific knowledge is relative to the methodology of verification.

This is a long way from Plato's cave, in which the prisoners see only shadows but reason can reach deeper to the true forms casting the shadows. These exist in some timeless place where there is no idea of process. It is also a long way from Aristotle's three pillars: causality, explanation, and metaphysics. For Aristotle, reason could explain the observations by placing them within some rational structure intrinsic to the whole of reality. For both Plato and Aristotle,

truth is metaphysical, it being a property of an idea that, while it might be only partially revealed in observations, is intrinsic to the idea. For science, the truth of an idea depends on the process of validating its truth. Since many processes might be used, there are many truths. Change the process and the truth may change.

Some might try to argue that a truth relative to its process of verification is no more solid than Rousseau's mental fantasies. This would be a grossly fallacious analogy. Rousseau specifically states that facts do not matter, whereas a scientific theory must show concordance with facts. What is open in science is the manner in which concordance is to be manifested. One might argue that this leaves open the possibility of positing operational requirements that are so loose that any theory could be validated. This argument is facetious because it presupposes scientific nihilism, a position rejected by serious scientists and demonstrated by their willingness to put aside the idols of the mind to discover mathematical conceptualizations of natural processes consistent with observations across time.

5.6 A New Role for Reason

Aristotle provides four causes as the basis for explanation of the physical world. Irrespective of the continuing appeal to causality, explanation remains ubiquitous and is perhaps the greatest impediment to meaningful scientific enquiry. Explanation makes the world intelligible by characterizing it via categories grasped by the intellect, thereby satisfying the emotional desire to give order to the physical world and comprehend the “why” of that order. Nature seemingly becomes accessible to the human intellect. The result is reason working *a posteriori* on observations or perhaps in the absence of observations (think of Rousseau) to construct a mental picture of the world. This would be a picture in terms of human physical concepts such as particles, gravity, force, etc. It would be a picture of Nature filtered through the idols of the tribe, seen in the reflection of “a false mirror, which, receiving rays irregularly, distorts and discolors the nature of things by mingling its own nature with it.”

Science has not abandoned reason; rather, the role of reason has changed. Scientific knowledge is constituted in a most pure form of reason, mathematics, but the truth of that knowledge is not ascertained directly by reason, nor is that knowledge required to conform to ordinary categories of intelligibility. In one sense, reason loses its lofty position because it cannot remain independent in its judgments; these must be tied to phenomena in well-defined ways. To put the matter more forcefully, reason is no longer trusted.

The Enlightenment, in the person of its two greatest philosophers, Hume and Kant, turns reason upon itself and exposes its limitations, at least in its pure form. When Maxwell speaks of discovering a method that allows the mind not to be “carried beyond the truth by a favorite hypothesis,” he is warning of the danger of unchecked reason, a warning given more forcefully by Hume, who, in the *Treatise*, asserts, “Reason is, and ought only to be the slave of the passions, and can never pretend to any other office than to serve and obey them.” [Hume,

1738] Whereas Maxwell is concerned about tilting one's reason in the direction of a favorite hypothesis owing to "that blindness to facts and rashness in assumption which a partial explanation encourages," Hume, with his usual flair for directness, states that reason is a servant of desire and therefore cannot be trusted as an arbiter of its own deliberations. One should not only be wary of blindness to the facts affecting explanations but also recognize that explanations may be constructed in such a way as to "serve and obey" the passions (again think of Rousseau). Consider two scientific protagonists who firmly believe in the products of their individual reason. We need not dig into the intricacies of their cobwebs. We need only test their claims, which can be done because they must each provide operational definitions in conjunction with their models.

Perhaps modernity has to some extent deprived reason of its lofty perch; however, it has also made reason more powerful in other ways. First, it has made an extraordinary move away from the immediate perceptions that were previously the basis for understanding the natural order. This entails a huge leap in creativity. Einstein writes, "Experience, of course, remains the sole criterion for the serviceability of mathematical constructions for physics, but the truly creative principle resides in mathematics." [Einstein, 1933] The veracity of a scientific model lies in experience, but its conception arises from the imagination, an imagination freed from the fetters of Euclidean geometry, linear time, certainty, causality, and other constraints of the past. Second, when confronting Nature, reason no longer is confined to groping through aimlessly collected data; instead, it views Nature through an experimental filter based upon its own needs. Third, science has abandoned the rational explanation of Nature, and reason no longer is stuck looking backwards in an attempt to explain the past; rather, its role is to foretell the future. Recall Reichenbach: "Observation informs us about the past and the present, reason foretells the future." To be able to predict the future puts great power into the hands of mankind because it facilitates the predictable transformation of Nature resulting from human action in the world. Science provides a "functional conception of knowledge."

5.7 Deterministic or Stochastic Models?

An advantage of a deterministic theory is that, assuming sufficient knowledge, there is no uncertainty in the evolution of the state of the system. In practice, measurements are not perfectly precise, so there is always uncertainty as to the value of any variable. This uncertainty does not undermine a deterministic epistemology; rather, it pertains to the actualization of the epistemology in the measurement process. One might anticipate increasingly precise measurements, to the point that measurement error would be negligible. This assumption vanishes with quantum theory, where, in principle, there is a hard limit.

According to the uncertainty principle, at any moment in time, the product of the uncertainties in position and momentum of a particle must exceed $h/4\pi$. The position and momentum can be measured separately without a limit on accuracy, but not jointly. According to the Copenhagen interpretation, the uncertainty

principle is intrinsic to human interaction with Nature, so that stochastic modeling in quantum mechanics is necessary. However, suppose Einstein is vindicated and hidden variables are found, so that a deterministic theory is sufficient relative to all known phenomena, or that the level of randomness is reduced. The new theory would be contingent, as are all scientific theories, awaiting new observations that might render it inadequate.

The fundamental point is that causality and determinism are metaphysical concepts. Recall Schrödinger's comment that causality is just "a characteristic of the way in which we regard Nature." For a scientific theory, the choice of a stochastic or deterministic model is pragmatic: Which gives better predictions?

Constraints are typically imposed on science by observational limitations. Since a model can only be verified to the extent that its symbols can be tied to observations, the ability to design and perform suitable experiments, including the availability of technology to make the desired measurements, is mandatory. Limitations on experimentation can result in limitations on the complexity or details of a theory. To be validated, a theory cannot exceed the experimentalist's ability to conceive and perform appropriate experiments. With the uncertainty theory, modern physics appears to have brought us beyond the situation where limitations on observation result only from insufficient experimental apparatus to a point where limitations are unsurpassable in principle.

Schrödinger states,

It really is the ultimate purpose of all schemes and models to serve as scaffolding for any observations that are at all conceivable.... There does not seem to be much sense in inquiring about the real existence of something, if one is convinced that the effect through which the thing would manifest itself, in case it existed, is certainly not observable. [Schrödinger, 1957]

Absent observable effects due to an object, the object is not a suitable subject for scientific inquiry.

We need not go to the uncertainty theory to appreciate Schrödinger's point. The inability to experience absolute simultaneity and other such absolutes plays a key role in Einstein's approach to relativity theory. He writes,

A further characterization of the theory of relativity is an epistemological point of view. In physics no concept is necessary or justifiable on an *a priori* basis. A concept acquires a right to existence solely through its obvious and unequivocal place in a chain of events relating to physical experiences. That is why the theory of relativity rejects concepts of absolute simultaneity, absolute speed, absolute acceleration, etc.; they can have no unequivocal link with experiences. Similarly, the notions of 'plane,' and 'straight line,' and the like, which form the basis of Euclidean geometry, had to be discarded. Every physical concept must

be defined in such a way that it can be used to determine in principle whether or not it fits the concrete case. [Einstein, 1993]

A second constraint on scientific theory imposed by observational limitations concerns the kind of mathematical models to be employed. If there is inherent uncertainty in the measurements relating to a model, then a deterministic model is limited in its ability to produce accurate predictions because phenomenal predictions tied to the model via its operational definitions will be affected by the uncertainty and therefore validation is problematic. Consequently, probabilistic models, taking uncertainty into account, are preferable. Whereas imprecise measurements always affect model validation, the uncertainty principle makes this problem intrinsic. This does not imply that deterministic models are no longer useful. In the classical setting, when measurement error is very small, it can be ignored. This is also true in the macroscopic world when it comes to quantum uncertainty because Planck's constant is very small and the uncertainty can be practically ignored.

Deterministic models may be suitable for simple physical systems not subject to consequential changes outside those internal to the system; however, they are rarely, if ever, satisfactory for modeling complex interactive physical systems subject to external variables outside the system, which are ubiquitous in biology. If a dynamical process is repeatedly observed and measurements made on some set of variables over time, one cannot expect the measurements to remain the same across the different trials because, even if one could somehow replicate the initial state of the variables for each trial, unless the process is completely isolated so that the variables being measured are affected by no others but themselves, its evolution will depend upon variables outside the set.

Like determinism interpreted as a world view, randomness is a metaphysical category that can neither be proved nor disproved by empirical observations. The assumption of a stochastic model is a scientific decision, not a metaphysical perspective. Andrey Kolmogorov, discoverer of the measure-theoretic approach to probability theory, writes, "The possibility of using, in the treatment of a real process, schemes of well-determined or of only stochastically definite processes stands in no relation to the question whether the real process is itself determined or random." [Kolmogorov, 1931] The "real process" is not a subject of scientific knowledge.